CHAPTER 4
Forces and Newton's Laws of Motion

Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. (Credit: Jin Jang)

Chapter Outline

4.1 Force

4.2 Newton's First Law of Motion: Inertia

4.3 Newton's Second Law of Motion

4.4 Newton's Third Law of Motion

INTRODUCTION Isaac Newton (1642–1727) was a natural philosopher; a great thinker who combined science and philosophy to try to explain the workings of nature on Earth and in the universe. His laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance period of history to the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. Drawing upon earlier work by scientists Galileo Galilei and Johannes Kepler, Newton's laws of motion allowed motion on Earth and in space to be predicted mathematically. In this chapter you will learn about force as well as Newton's first, second, and third laws of motion.

4.1 Force

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Differentiate between force, net force, and dynamics
- Draw a free-body diagram

Section Key Terms

dynamics	external force	force
free-body diagram	net external force	net force

Defining Force and Dynamics

Force is the cause of motion, and motion draws our attention. Motion itself can be beautiful, such as a dolphin jumping out of the water, the flight of a bird, or the orbit of a satellite. The study of motion is called kinematics, but kinematics describes only the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws describe the way objects speed up, slow down, stay in motion, and interact with other objects. They are also universal laws: they apply everywhere on Earth as well as in space.

A force pushes or pulls an object. The object being moved by a force could be an inanimate object, a table, or an animate object, a person. The pushing or pulling may be done by a person, or even the gravitational pull of Earth. Forces have different magnitudes and directions; this means that some forces are stronger than others and can act in different directions. For example, a cannon exerts a strong force on the cannonball that is launched into the air. In contrast, a mosquito landing on your arm exerts only a small force on your arm.

When multiple forces act on an object, the forces combine. Adding together all of the forces acting on an object gives the total force, or **net force**. An **external force** is a force that acts on an object within the system *from outside* the system. This type of force is different than an internal force, which acts between two objects that are both within the system. **The net external force** combines these two definitions; it is the total combined external force. We discuss further details about net force, external force, and net external force in the coming sections.

In mathematical terms, two forces acting in opposite directions have opposite *signs* (positive or negative). By convention, the negative sign is assigned to any movement to the left or downward. If two forces pushing in opposite directions are added together, the larger force will be somewhat canceled out by the smaller force pushing in the opposite direction. It is important to be consistent with your chosen coordinate system within a problem; for example, if negative values are assigned to the downward direction for velocity, then distance, force, and acceleration should also be designated as being negative in the downward direction.

Free-Body Diagrams and Examples of Forces

For our first example of force, consider an object hanging from a rope. This example gives us the opportunity to introduce a useful tool known as a **free-body diagram**. A free-body diagram represents the object being acted upon—that is, the free body—as a single point. Only the forces acting *on* the body (that is, external forces) are shown and are represented by vectors (which are drawn as arrows). These forces are the only ones shown because only external forces acting on the body affect its motion. We can ignore any internal forces within the body because they cancel each other out, as explained in the section on Newton's third law of motion. Free-body diagrams are very useful for analyzing forces acting on an object.

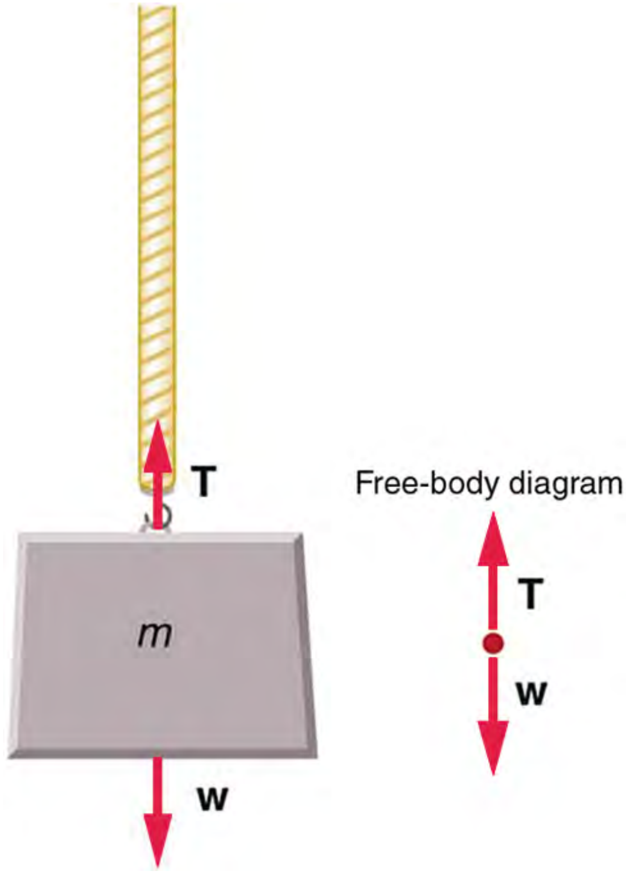

Figure 4.2 An object of mass, *m*, is held up by the force of tension.

Figure 4.2 shows the force of tension in the rope acting in the upward direction, opposite the force of gravity. The forces are indicated in the free-body diagram by an arrow pointing up, representing tension, and another arrow pointing down, representing gravity. In a free-body diagram, the lengths of the arrows show the relative magnitude (or strength) of the forces. Because forces are vectors, they add just like other vectors. Notice that the two arrows have equal lengths in Figure 4.2, which means that the forces of tension and weight are of equal magnitude. Because these forces of equal magnitude act in opposite directions, they are perfectly balanced, so they add together to give a net force of zero.

Not all forces are as noticeable as when you push or pull on an object. Some forces act without physical contact, such as the pull of a magnet (in the case of magnetic force) or the gravitational pull of Earth (in the case of gravitational force).

In the next three sections discussing Newton's laws of motion, we will learn about three specific types of forces: friction, the normal force, and the gravitational force. To analyze situations involving forces, we will create free-body diagrams to organize the framework of the mathematics for each individual situation.

TIPS FOR SUCCESS

Correctly drawing and labeling a free-body diagram is an important first step for solving a problem. It will help you visualize the problem and correctly apply the mathematics to solve the problem.

Check Your Understanding

1. What is kinematics?
   a. Kinematics is the study of motion.
   b. Kinematics is the study of the cause of motion.
   c. Kinematics is the study of dimensions.
   d. Kinematics is the study of atomic structures.

2. Do two bodies have to be in physical contact to exert a force upon one another?

a. No, the gravitational force is a field force and does not require physical contact to exert a force.
b. No, the gravitational force is a contact force and does not require physical contact to exert a force.
c. Yes, the gravitational force is a field force and requires physical contact to exert a force.
d. Yes, the gravitational force is a contact force and requires physical contact to exert a force.

3. What kind of physical quantity is force?

a. Force is a scalar quantity.
b. Force is a vector quantity.
c. Force is both a vector quantity and a scalar quantity.
d. Force is neither a vector nor a scalar quantity.

4. Which forces can be represented in a free-body diagram?

a. Internal forces
b. External forces
c. Both internal and external forces
d. A body that is not influenced by any force

4.2 Newton's First Law of Motion: Inertia

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's first law and friction, and
- Discuss the relationship between mass and inertia.

Section Key Terms

friction	inertia	law of inertia
mass	Newton's first law of motion	system

Newton's First Law and Friction

Newton's first law of motion states the following:

1. A body at rest tends to remain at rest.
2. A body in motion tends to remain in motion at a constant velocity unless acted on by a net external force. (Recall that *constant velocity* means that the body moves in a straight line and at a constant speed.)

At first glance, this law may seem to contradict your everyday experience. You have probably noticed that a moving object will usually slow down and stop unless some effort is made to keep it moving. The key to understanding why, for example, a sliding box slows down (seemingly on its own) is to first understand that a net external force acts on the box to make the box slow down. Without this net external force, the box would continue to slide at a constant velocity (as stated in Newton's first law of motion). What force acts on the box to slow it down? This force is called **friction**. Friction is an external force that acts opposite to the direction of motion (see Figure 4.3). Think of friction as a resistance to motion that slows things down.

Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it lifts the puck slightly, so the puck experiences very little friction as it moves over the surface. With friction almost eliminated, the puck glides along with very little change in speed. On a frictionless surface, the puck would experience no net external force (ignoring air resistance, which is also a form of friction). Additionally, if we know enough about friction, we can accurately predict how quickly objects will slow down.

Now let's think about another example. A man pushes a box across a floor at constant velocity by applying a force of +50 N. (The positive sign indicates that, by convention, the direction of motion is to the right.) What is the force of friction that opposes the motion? The force of friction must be –50 N. Why? According to Newton's first law of motion, any object moving at constant velocity has no net external force acting upon it, which means that the sum of the forces acting on the object must be zero. The mathematical way to say that no net external force acts on an object is $\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$. So if the man applies +50 N of force, then the force of friction must be –50 N for the two forces to add up to zero (that is, for the two forces to *cancel* each other). Whenever you encounter the phrase *at constant velocity*, Newton's first law tells you that the net external force is zero.

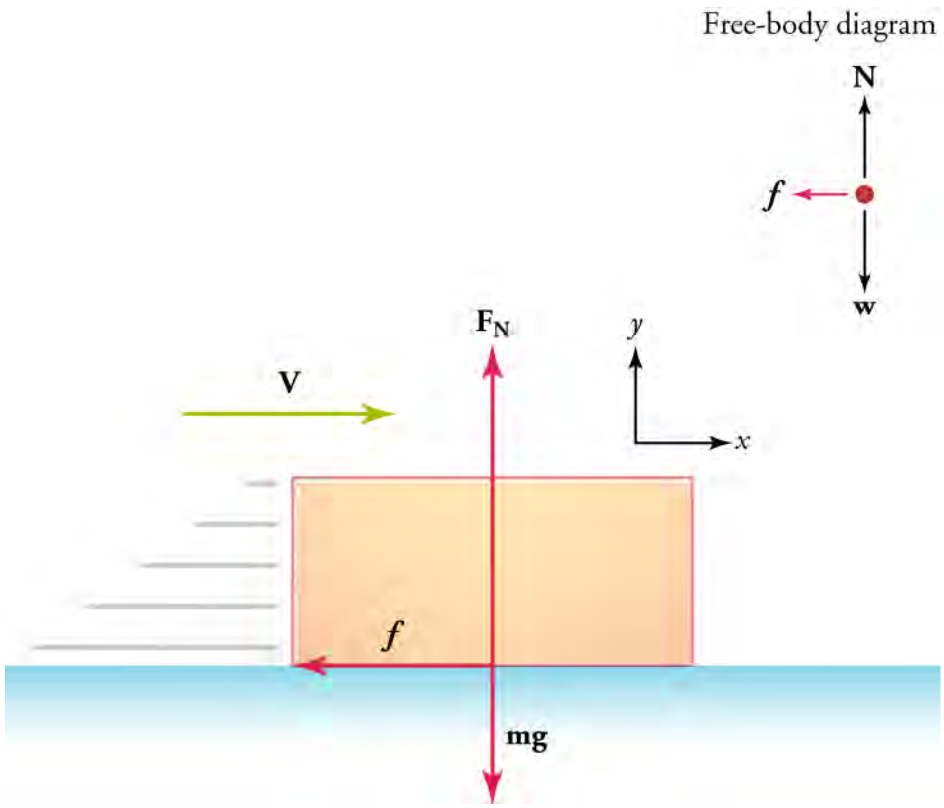

Figure 4.3 For a box sliding across a floor, friction acts in the direction opposite to the velocity.

The force of friction depends on two factors: the coefficient of friction and the normal force. For any two surfaces that are in contact with one another, the coefficient of friction is a constant that depends on the nature of the surfaces. The normal force is the force exerted by a surface that pushes on an object in response to gravity pulling the object down. In equation form, the force of friction is

$$\mathbf{f} = \mu \mathbf{N}, \tag{4.1}$$

where μ is the coefficient of friction and **N** is the normal force. (The coefficient of friction is discussed in more detail in another chapter, and the normal force is discussed in more detail in the section *Newton's Third Law of Motion*.)

Recall from the section on Force that a net external force acts from outside on the object of interest. A more precise definition is that it acts on the **system** of interest. A system is one or more objects that you choose to study. It is important to define the system at the beginning of a problem to figure out which forces are external and need to be considered, and which are internal and can be ignored.

For example, in Figure 4.4 (a), two children push a third child in a wagon at a constant velocity. The system of interest is the wagon plus the small child, as shown in part (b) of the figure. The two children behind the wagon exert external forces on this system ($\mathbf{F}_1$, $\mathbf{F}_2$). Friction *f* acting at the axles of the wheels and at the surface where the wheels touch the ground two other external forces acting on the system. Two more external forces act on the system: the weight **W** of the system pulling down and the normal force **N** of the ground pushing up. Notice that the wagon is not accelerating vertically, so Newton's first law tells us that the normal force balances the weight. Because the wagon is moving forward at a constant velocity, the force of friction must have the same strength as the sum of the forces applied by the two children.

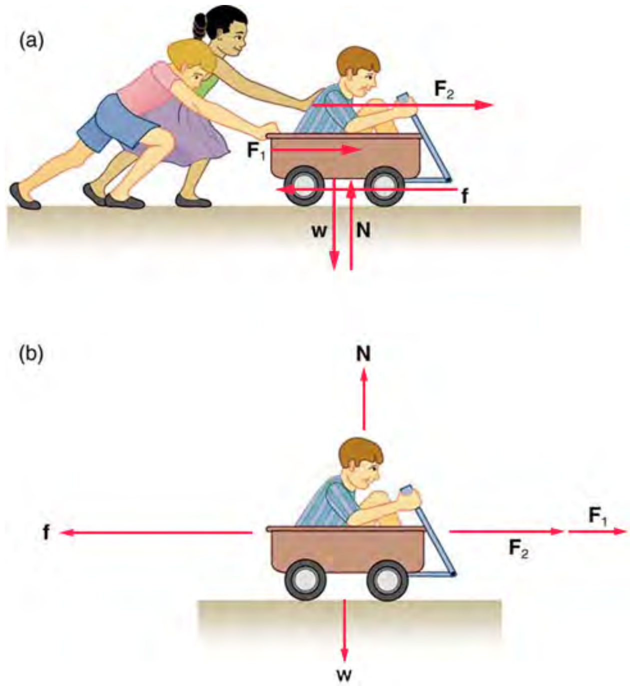

Figure 4.4 (a) The wagon and rider form a *system* that is acted on by external forces. (b) The two children pushing the wagon and child provide two external forces. Friction acting at the wheel axles and on the surface of the tires where they touch the ground provide an external force that act against the direction of motion. The weight **W** and the normal force **N** from the ground are two more external forces acting on the system. All external forces are represented in the figure by arrows. All of the external forces acting on the system add together, but because the wagon moves at a constant velocity, all of the forces must add up to zero.

Mass and Inertia

Inertia is the tendency for an object at rest to remain at rest, or for a moving object to remain in motion in a straight line with constant speed. This key property of objects was first described by Galileo. Later, Newton incorporated the concept of inertia into his first law, which is often referred to as the **law of inertia**.

As we know from experience, some objects have more inertia than others. For example, changing the motion of a large truck is more difficult than changing the motion of a toy truck. In fact, the inertia of an object is proportional to the mass of the object. **Mass** is a measure of the amount of matter (or *stuff*) in an object. The quantity or amount of matter in an object is determined by the number and types of atoms the object contains. Unlike weight (which changes if the gravitational force changes), mass does not depend on gravity. The mass of an object is the same on Earth, in orbit, or on the surface of the moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so mass is usually not determined this way. Instead, the mass of an object is determined by comparing it with the standard kilogram. Mass is therefore expressed in kilograms.

TIPS FOR SUCCESS

In everyday language, people often use the terms *weight* and *mass* interchangeably—but this is not correct. Weight is actually a force. (We cover this topic in more detail in the section *Newton's Second Law of Motion*.)

WATCH PHYSICS

Newton's First Law of Motion

This video contrasts the way we thought about motion and force in the time before Galileo's concept of inertia and Newton's first law of motion with the way we understand force and motion now.

[Click to view content (https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)](https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)

GRASP CHECK

Before we understood that objects have a tendency to maintain their velocity in a straight line unless acted upon by a net force, people thought that objects had a tendency to stop on their own. This happened because a specific force was not yet understood. What was that force?

a. Gravitational force
b. Electrostatic force
c. Nuclear force
d. Frictional force

Virtual Physics

Forces and Motion—Basics

In this simulation, you will first explore net force by placing blue people on the left side of a tug-of-war rope and red people on the right side of the rope (by clicking people and dragging them with your mouse). Experiment with changing the number and size of people on each side to see how it affects the outcome of the match and the net force. Hit the "Go!" button to start the match, and the "reset all" button to start over.

Next, click on the Friction tab. Try selecting different objects for the person to push. Slide the *applied force* button to the right to apply force to the right, and to the left to apply force to the left. The force will continue to be applied as long as you hold down the button. See the arrow representing friction change in magnitude and direction, depending on how much force you apply. Try increasing or decreasing the friction force to see how this change affects the motion.

Click to view content (https://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html)

GRASP CHECK

Click on the tab for the *Acceleration Lab and check the Sum of Forces* option. Push the box to the right and then release. Notice which direction the sum of forces arrow points after the person stops pushing the box and lets it continue moving to the right on its own. At this point, in which direction is the net force, the sum of forces, pointing? Why?

a. The net force acts to the right because the applied external force acted to the right.
b. The net force acts to the left because the applied external force acted to the left.
c. The net force acts to the right because the frictional force acts to the right.
d. The net force acts to the left because the frictional force acts to the left.

Check Your Understanding

5. What does Newton's first law state?

a. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant acceleration unless acted on by a net external force.
b. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant velocity unless acted on by a net external force.
c. The rate of change of momentum of a body is directly proportional to the external force applied to the body.
d. The rate of change of momentum of a body is inversely proportional to the external force applied to the body.

6. According to Newton's first law, a body in motion tends to remain in motion at a constant velocity. However, when you slide an object across a surface, the object eventually slows down and stops. Why?

a. The object experiences a frictional force exerted by the surface, which opposes its motion.
b. The object experiences the gravitational force exerted by Earth, which opposes its motion
c. The object experiences an internal force exerted by the body itself, which opposes its motion.
d. The object experiences a pseudo-force from the body in motion, which opposes its motion.

7. What is inertia?
   a. Inertia is an object's tendency to maintain its mass.
   b. Inertia is an object's tendency to remain at rest.
   c. Inertia is an object's tendency to remain in motion
   d. Inertia is an object's tendency to remain at rest or, if moving, to remain in motion.

8. What is mass? What does it depend on?
   a. Mass is the weight of an object, and it depends on the gravitational force acting on the object.
   b. Mass is the weight of an object, and it depends on the number and types of atoms in the object.
   c. Mass is the quantity of matter contained in an object, and it depends on the gravitational force acting on the object.
   d. Mass is the quantity of matter contained in an object, and it depends on the number and types of atoms in the object.

4.3 Newton's Second Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's second law, both verbally and mathematically
- Use Newton's second law to solve problems

Section Key Terms

freefall	Newton's second law of motion	weight

Describing Newton's Second Law of Motion

Newton's first law considered bodies at rest or bodies in motion at a constant velocity. The other state of motion to consider is when an object is moving with a changing velocity, which means a change in the speed and/or the direction of motion. This type of motion is addressed by **Newton's second law of motion**, which states how force causes changes in motion. Newton's second law of motion is used to calculate what happens in situations involving forces and motion, and it shows the mathematical relationship between force, mass, and *acceleration*. Mathematically, the second law is most often written as

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \text{ or } \Sigma\mathbf{F} = m\mathbf{a}, \tag{4.2}$$

where $\mathbf{F}_{\text{net}}$ (or $\sum\mathbf{F}$) is the net external force, m is the mass of the system, and $\mathbf{a}$ is the acceleration. Note that $\mathbf{F}_{\text{net}}$ and $\sum\mathbf{F}$ are the same because the net external force is the sum of all the external forces acting on the system.

First, what do we mean by *a change in motion*? A change in motion is simply a change in velocity: the speed of an object can become slower or faster, the direction in which the object is moving can change, or both of these variables may change. A change in velocity means, by definition, that an acceleration has occurred. Newton's first law says that only a nonzero net external force can cause a change in motion, so a net external force must cause an acceleration. Note that acceleration can refer to slowing down or to speeding up. Acceleration can also refer to a change in the direction of motion with no change in speed, because acceleration is the change in velocity divided by the time it takes for that change to occur, *and* velocity is defined by speed *and* direction.

From the equation $\mathrm{F}_{\text{net}} = m\mathrm{a}$, we see that force is directly proportional to both mass and acceleration, which makes sense. To accelerate two objects from rest to the same velocity, you would expect more force to be required to accelerate the more massive object. Likewise, for two objects of the same mass, applying a greater force to one would accelerate it to a greater velocity.

Now, let's rearrange Newton's second law to solve for acceleration. We get

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m} \text{ or } \mathbf{a} = \frac{\Sigma\mathbf{F}}{m}. \tag{4.3}$$

In this form, we can see that acceleration is directly proportional to force, which we write as

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{4.4}$$

where the symbol $\propto$ means *proportional to*.

This proportionality mathematically states what we just said in words: acceleration is directly proportional to the net external force. When two variables are directly proportional to each other, then if one variable doubles, the other variable must double. Likewise, if one variable is reduced by half, the other variable must also be reduced by half. In general, when one variable is multiplied by a number, the other variable is also multiplied by the same number. It seems reasonable that the acceleration of a system should be directly proportional to and in the same direction as the net external force acting on the system. An object experiences greater acceleration when acted on by a greater force.

It is also clear from the equation $\mathbf{a} = \mathbf{F}_{\text{net}}/m$ that acceleration is inversely proportional to mass, which we write as

$$\mathbf{a} \propto \frac{1}{m}. \tag{4.5}$$

Inversely proportional means that if one variable is multiplied by a number, the other variable must be *divided* by the same number. Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. This relationship is illustrated in Figure 4.5, which shows that a given net external force applied to a basketball produces a much greater acceleration than when applied to a car.

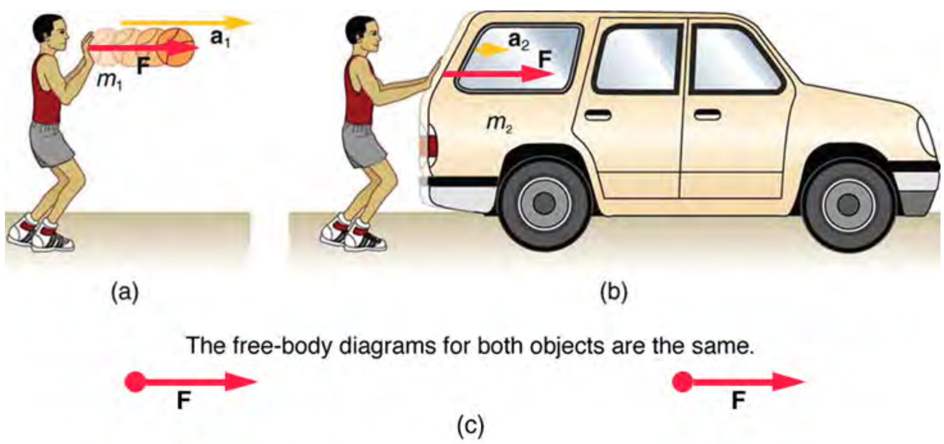

Figure 4.5 The same force exerted on systems of different masses produces different accelerations. (a) A boy pushes a basketball to make a pass. The effect of gravity on the ball is ignored. (b) The same boy pushing with identical force on a stalled car produces a far smaller acceleration (friction is negligible). Note that the free-body diagrams for the ball and for the car are identical, which allows us to compare the two situations.

Applying Newton's Second Law

Before putting Newton's second law into action, it is important to consider units. The equation $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units of mass, length, and time (recall that acceleration has units of length divided by time squared). The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of $1\ \text{m/s}^2$. That is, because $\mathbf{F}_{\text{net}} = m\mathbf{a}$, we have

$$1\ \text{N} = 1\ \text{kg} \times 1\text{m/s}^2 = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}. \tag{4.6}$$

One of the most important applications of Newton's second law is to calculate **weight** (also known as the gravitational force), which is usually represented mathematically as $\mathbf{W}$. When people talk about gravity, they don't always realize that it is an acceleration. When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that the net external force acting on an object is responsible for the acceleration of the object. If air resistance is negligible, the net external force on a falling object is only the gravitational force (i.e., the weight of the object).

Weight can be represented by a vector because it has a direction. Down is defined as the direction in which gravity pulls, so weight is normally considered a downward force. By using Newton's second law, we can figure out the equation for weight.

Consider an object with mass m falling toward Earth. It experiences only the force of gravity (i.e., the gravitational force or weight), which is represented by W. Newton's second law states that $\mathbf{F}_{\text{net}} = m\mathbf{a}$. Because the only force acting on the object is the gravitational force, we have $\mathbf{F}_{\text{net}} = \mathbf{W}$. We know that the acceleration of an object due to gravity is $\mathbf{g}$, so we have $\mathbf{a} = \mathbf{g}$. Substituting these two expressions into Newton's second law gives

$$\mathbf{W} = m\mathbf{g}. \tag{4.7}$$

This is the equation for weight—the gravitational force on a mass m. On Earth, $\mathbf{g} = 9.80 \text{ m/s}^2$, so the weight (disregarding for now the direction of the weight) of a 1.0-kg object on Earth is

$$\mathbf{W} = m\mathbf{g} = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}. \tag{4.8}$$

Although most of the world uses newtons as the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Recall that although gravity acts downward, it can be assigned a positive or negative value, depending on what the positive direction is in your chosen coordinate system. Be sure to take this into consideration when solving problems with weight. When the downward direction is taken to be negative, as is often the case, acceleration due to gravity becomes $\mathbf{g} = -9.8 \text{ m/s}^2$.

When the net external force on an object is its weight, we say that it is in **freefall**. In this case, the only force acting on the object is the force of gravity. On the surface of Earth, when objects fall downward toward Earth, they are never truly in freefall because there is always some upward force due to air resistance that acts on the object (and there is also the buoyancy force of air, which is similar to the buoyancy force in water that keeps boats afloat).

Gravity varies slightly over the surface of Earth, so the weight of an object depends very slightly on its location on Earth. Weight varies dramatically away from Earth's surface. On the moon, for example, the acceleration due to gravity is only 1.67 m/s^2. Because weight depends on the force of gravity, a 1.0-kg mass weighs 9.8 N on Earth and only about 1.7 N on the moon.

It is important to remember that weight and mass are very different, although they are closely related. Mass is the quantity of matter (how much *stuff*) in an object and does not vary, but weight is the gravitational force on an object and is proportional to the force of gravity. It is easy to confuse the two, because our experience is confined to Earth, and the weight of an object is essentially the same no matter where you are on Earth. Adding to the confusion, the terms mass and weight are often used interchangeably in everyday language; for example, our medical records often show our weight in kilograms, but never in the correct unit of newtons.

Snap Lab

Mass and Weight

In this activity, you will use a scale to investigate mass and weight.

- 1 bathroom scale
- 1 table

1. What do bathroom scales measure?
2. When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled.
3. The springs provide a measure of your weight (provided you are not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is now divided by 9.80 to give a reading in kilograms, which is a of mass. The scale detects weight but is calibrated to display mass.
4. If you went to the moon and stood on your scale, would it detect the same *mass* as it did on Earth?

GRASP CHECK

While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why?

a. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
b. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction opposite to your weight.
c. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
d. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on

you that acts in the direction opposite to your weight.

TIPS FOR SUCCESS

Only *net external force* impacts the acceleration of an object. If more than one force acts on an object and you calculate the acceleration by using only one of these forces, you will not get the correct acceleration for that object.

WATCH PHYSICS

Newton's Second Law of Motion

This video reviews Newton's second law of motion and how net external force and acceleration relate to one another and to mass. It also covers units of force, mass, and acceleration, and reviews a worked-out example.

Click to view content (https://www.khanacademy.org/embed_video?v=ou9YMWlJgkE)

GRASP CHECK

True or False—If you want to reduce the acceleration of an object to half its original value, then you would need to reduce the net external force by half.

a. True
b. False

WORKED EXAMPLE

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N parallel to the ground. The mass of the mower is 240 kg. What is its acceleration?

Figure 4.6

Strategy

Because $\mathbf{F}_{\mathbf{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law: $\mathbf{F}_{\mathbf{net}} = m\mathbf{a}$.

Solution

Solving Newton's second law for the acceleration, we find that the magnitude of the acceleration, $\mathbf{a}$, is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$. Entering the given values for net external force and mass gives

$$\mathbf{a} = \frac{51\ \text{N}}{240\ \text{kg}} \tag{4.9}$$

Inserting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$\mathbf{a} = \frac{51\ \text{kg} \cdot \text{m/s}^2}{240\ \text{kg}} = 0.21\ \text{m/s}^2. \tag{4.10}$$

Discussion

The acceleration is in the same direction as the net external force, which is parallel to the ground and to the right. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion, because we are given that the net external force is in the direction in which the person pushes. Also, the vertical forces must cancel if there is no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is reasonable for a person pushing a mower; the mower's speed must increase by 0.21 m/s every second, which is possible. The time during which the mower accelerates would not be very long because the person's top speed would soon be reached. At this point, the person could push a little less hard, because he only has to overcome friction.

WORKED EXAMPLE

What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on humans at high accelerations. Rocket sleds consisted of a platform mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust, **T**, for the four-rocket propulsion system shown below. The sled's initial acceleration is $49\ \text{m/s}^2$, the mass of the system is 2,100 kg, and the force of friction opposing the motion is 650 N.

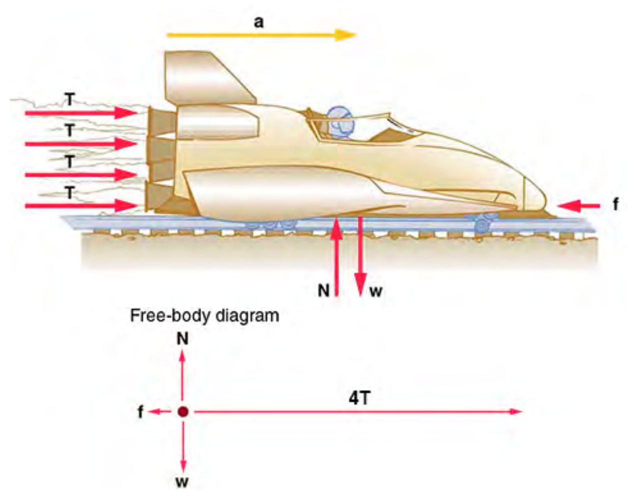

Figure 4.7

Strategy

The system of interest is the rocket sled. Although forces act vertically on the system, they must cancel because the system does not accelerate vertically. This leaves us with only horizontal forces to consider. We'll assign the direction to the right as the positive direction. See the free-body diagram in Figure 4.8.

Solution

We start with Newton's second law and look for ways to find the thrust **T** of the engines. Because all forces and acceleration are along a line, we need only consider the magnitudes of these quantities in the calculations. We begin with

$$\mathbf{F}_{\text{net}} = m\mathbf{a}, \tag{4.11}$$

where $\mathbf{F}_{\text{net}}$ is the net external force in the horizontal direction. We can see from Figure 4.8 that the engine thrusts are in the same direction (which we call the positive direction), whereas friction opposes the thrust. In equation form, the net external force is

$$\mathbf{F}_{\text{net}} = 4\mathbf{T} - \mathbf{f}. \tag{4.12}$$

Newton's second law tells us that $\mathbf{F}_{\text{net}} = m\mathbf{a}$, so we get

$$m\mathbf{a} = 4\mathbf{T} - \mathbf{f}. \tag{4.13}$$

After a little algebra, we solve for the total thrust 4**T**:

$$4\mathbf{T} = m\mathbf{a} + \mathbf{f}, \tag{4.14}$$

which means that the individual thrust is

$$\mathbf{T} = \frac{m\mathbf{a} + \mathbf{f}}{4}. \tag{4.15}$$

Inserting the known values yields

$$\mathbf{T} = \frac{(2100\text{ kg})(49\text{ m/s}^2) + 650\text{ N}}{4} = 2.6 \times 10^4\text{ N}. \tag{4.16}$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and to test the apparatus designed to protect fighter pilots in emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 **g**. (Recall that g, the acceleration due to gravity, is 9.80 m/s^2. An acceleration of 45 **g** is $45 \times 9.80\text{ m/s}^2$, which is approximately 440 m/s^2.) Living subjects are no longer used, and land speeds of 10,000 km/h have now been obtained with rocket sleds. In this example, as in the preceding example, the system of interest is clear. We will see in later examples that choosing the system of interest is crucial—and that the choice is not always obvious.

Practice Problems

9. If 1 N is equal to 0.225 lb, how many pounds is 5 N of force?
   a. 0.045 lb
   b. 1.125 lb
   c. 2.025 lb
   d. 5.000 lb

10. How much force needs to be applied to a 5-kg object for it to accelerate at 20 m/s^2?
   a. 1 N
   b. 10 N
   c. 100 N
   d. 1,000 N

Check Your Understanding

11. What is the mathematical statement for Newton's second law of motion?
   a. $F = ma$
   b. $F = 2ma$
   c. $F = \frac{m}{a}$
   d. $F = ma^2$

12. Newton's second law describes the relationship between which quantities?
   a. Force, mass, and time
   b. Force, mass, and displacement
   c. Force, mass, and velocity
   d. Force, mass, and acceleration

13. What is acceleration?
   a. Acceleration is the rate at which displacement changes.
   b. Acceleration is the rate at which force changes.
   c. Acceleration is the rate at which velocity changes.

d. Acceleration is the rate at which mass changes.

4.4 Newton's Third Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's third law, both verbally and mathematically
- Use Newton's third law to solve problems

Section Key Terms

Newton's third law of motion	normal force	tension	thrust

Describing Newton's Third Law of Motion

If you have ever stubbed your toe, you have noticed that although your toe initiates the impact, the surface that you stub it on exerts a force back on your toe. Although the first thought that crosses your mind is probably "ouch, that hurt" rather than "this is a great example of Newton's third law," both statements are true.

This is exactly what happens whenever one object exerts a force on another—each object experiences a force that is the same strength as the force acting on the other object but that acts in the opposite direction. Everyday experiences, such as stubbing a toe or throwing a ball, are all perfect examples of Newton's third law in action.

Newton's third law of motion states that whenever a first object exerts a force on a second object, the first object experiences a force equal in magnitude but opposite in direction to the force that it exerts.

Newton's third law of motion tells us that forces always occur in pairs, and one object cannot exert a force on another without experiencing the same strength force in return. We sometimes refer to these force pairs as *action-reaction* pairs, where the force exerted is the action, and the force experienced in return is the reaction (although which is which depends on your point of view).

Newton's third law is useful for figuring out which forces are external to a system. Recall that identifying external forces is important when setting up a problem, because the external forces must be added together to find the net force.

We can see Newton's third law at work by looking at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 4.8. She pushes against the pool wall with her feet and accelerates in the direction opposite to her push. The wall has thus exerted on the swimmer a force of equal magnitude but in the direction opposite that of her push. You might think that two forces of equal magnitude but that act in opposite directions would cancel, *but they do not because they act on different systems.*

In this case, there are two different systems that we could choose to investigate: the swimmer or the wall. If we choose the swimmer to be the system of interest, as in the figure, then $\mathrm{F}_{\text{wall on feet}}$ is an external force on the swimmer and affects her motion. Because acceleration is in the same direction as the net external force, the swimmer moves in the direction of $\mathrm{F}_{\text{wall on feet}}$. Because the swimmer is our system (or object of interest) and not the wall, we do not need to consider the force $\mathrm{F}_{\text{feet on wall}}$ because it originates *from* the swimmer rather than *acting on* the swimmer. Therefore, $\mathrm{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathrm{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to the direction in which she wants to move.

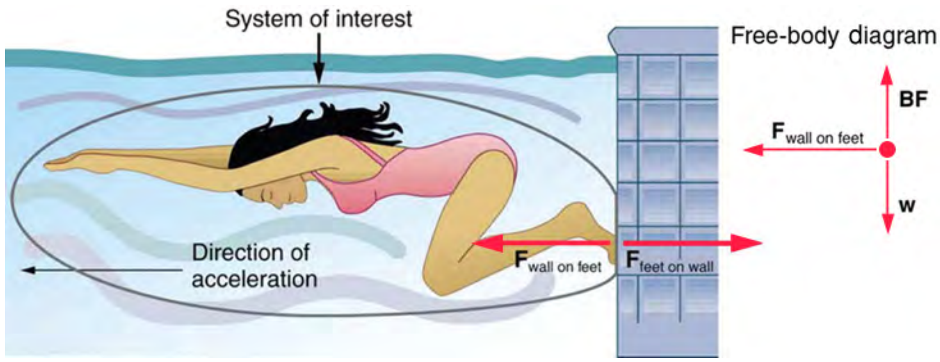

Figure 4.8 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means that the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition is the result of Newton's third law of motion, which dictates that the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but that acts in the direction opposite to the force that the swimmer exerts on the wall.

Other examples of Newton's third law are easy to find. As a teacher paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force in the forward direction on the teacher that causes him to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the car's wheels in reaction to the car's wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward.

Another example is the force of a baseball as it makes contact with the bat. Helicopters create lift by pushing air down, creating an upward reaction force. Birds fly by exerting force on air in the direction opposite that in which they wish to fly. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself forward in the water by ejecting water backward through a funnel in its body, which is similar to how a jet ski is propelled. In these examples, the octopus or jet ski push the water backward, and the water, in turn, pushes the octopus or jet ski forward.

Applying Newton's Third Law

Forces are classified and given names based on their source, how they are transmitted, or their effects. In previous sections, we discussed the forces called *push*, *weight*, and *friction*. In this section, applying Newton's third law of motion will allow us to explore three more forces: the **normal force**, **tension**, and **thrust**. However, because we haven't yet covered vectors in depth, we'll only consider one-dimensional situations in this chapter. Another chapter will consider forces acting in two dimensions.

The gravitational force (or weight) acts on objects at all times and everywhere on Earth. We know from Newton's second law that a net force produces an acceleration; so, why is everything not in a constant state of freefall toward the center of Earth? The answer is the normal force. The normal force is the force that a surface applies to an object to support the weight of that object; it acts perpendicular to the surface upon which the object rests. If an object on a flat surface is not accelerating, the net external force is zero, and the normal force has the same magnitude as the weight of the system but acts in the opposite direction. In equation form, we write that

$$\mathbf{N} = m\mathbf{g}. \tag{4.17}$$

Note that this equation is only true for a horizontal surface.

The word *tension* comes from the Latin word meaning *to stretch*. Tension is the force along the length of a flexible connector, such as a string, rope, chain, or cable. Regardless of the type of connector attached to the object of interest, one must remember that the connector can only pull (or *exert tension*) in the direction *parallel* to its length. Tension is a pull that acts parallel to the connector, and that acts in opposite directions at the two ends of the connector. This is possible because a flexible connector is simply a long series of action-reaction forces, except at the two ends where outside objects provide one member of the action-reaction forces.

Consider a person holding a mass on a rope, as shown in Figure 4.9.

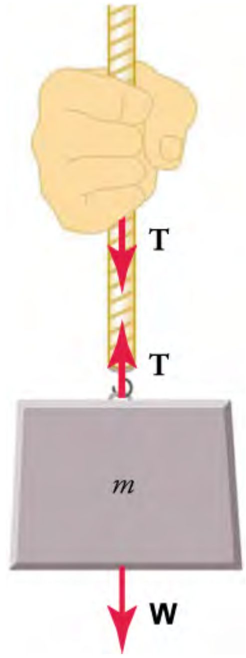

Figure 4.9 When a perfectly flexible connector (one requiring no force to bend it) such as a rope transmits a force **T**, this force must be parallel to the length of the rope, as shown. The pull that such a flexible connector exerts is a tension. Note that the rope pulls with equal magnitude force but in opposite directions to the hand and to the mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that transmits forces of equal magnitude between the two objects but that act in opposite directions.

Tension in the rope must equal the weight of the supported mass, as we can prove by using Newton's second law. If the 5.00 kg mass in the figure is stationary, then its acceleration is zero, so $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight **W** and the tension **T** supplied by the rope. Summing the external forces to find the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{T} - \mathbf{W} = 0, \tag{4.18}$$

where **T** and **W** are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. By substituting $m\mathbf{g}$ for $\mathbf{F}_{\text{net}}$ and rearranging the equation, the tension equals the weight of the supported mass, just as you would expect

$$\mathbf{T} = \mathbf{W} = m\mathbf{g}. \tag{4.19}$$

For a 5.00-kg mass (neglecting the mass of the rope), we see that

$$\mathbf{T} = m\mathbf{g} = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}. \tag{4.20}$$

Another example of Newton's third law in action is thrust. Rockets move forward by expelling gas backward at a high velocity. This means that the rocket exerts a large force backward on the gas in the rocket combustion chamber, and the gas, in turn, exerts a large force forward on the rocket in response. This reaction force is called *thrust*.

TIPS FOR SUCCESS

A common misconception is that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can expel exhaust gases more easily.

LINKS TO PHYSICS

Math: Problem-Solving Strategy for Newton's Laws of Motion

The basics of problem solving, presented earlier in this text, are followed here with specific strategies for applying Newton's laws of motion. These techniques also reinforce concepts that are useful in many other areas of physics.

First, identify the physical principles involved. If the problem involves forces, then Newton's laws of motion are involved, and it is important to draw a careful sketch of the situation. An example of a sketch is shown in Figure 4.10. Next, as in Figure 4.10, use vectors to represent all forces. Label the forces carefully, and make sure that their lengths are proportional to the magnitude of the forces and that the arrows point in the direction in which the forces act.

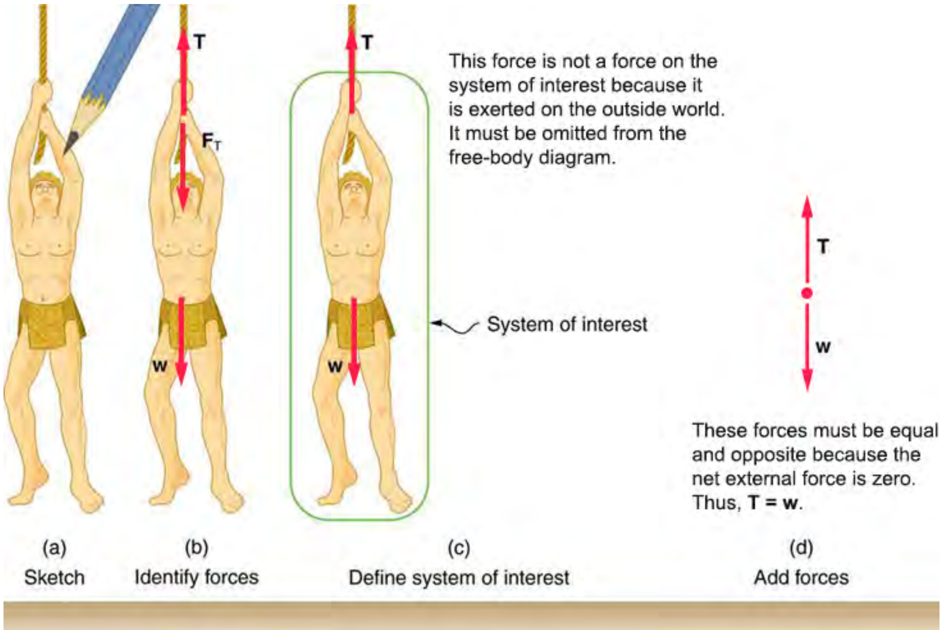

Figure 4.10 (a) A sketch of Tarzan hanging motionless from a vine. (b) Arrows are used to represent all forces. **T** is the tension exerted on Tarzan by the vine, $\mathbf{F}_\mathrm{T}$ is the force exerted on the vine by Tarzan, and **W** is Tarzan's weight (i.e., the force exerted on Tarzan by Earth's gravity). All other forces, such as a nudge of a breeze, are assumed to be negligible. (c) Suppose we are given Tarzan's mass and asked to find the tension in the vine. We define the system of interest as shown and draw a free-body diagram, as shown in (d). $\mathbf{F}_\mathrm{T}$ is no longer shown because it does not act on the system of interest; rather, $\mathbf{F}_\mathrm{T}$ acts on the outside world. (d) The free-body diagram shows only the external forces acting on Tarzan. For these to sum to zero, we must have $\mathbf{T} = \mathbf{W}$.

Next, make a list of knowns and unknowns and assign variable names to the quantities given in the problem. Figure out which variables need to be calculated; these are the unknowns. Now carefully define the system: which objects are of interest for the problem. This decision is important, because Newton's second law involves only external forces. Once the system is identified, it's possible to see which forces are external and which are internal (see Figure 4.10).

If the system acts on an object outside the system, then you know that the outside object exerts a force of equal magnitude but in the opposite direction on the system.

A diagram showing the system of interest and all the external forces acting on it is called a free-body diagram. Only external forces are shown on free-body diagrams, not acceleration or velocity. Figure 4.10 shows a free-body diagram for the system of interest.

After drawing a free-body diagram, apply Newton's second law to solve the problem. This is done in Figure 4.10 for the case of Tarzan hanging from a vine. When external forces are clearly identified in the free-body diagram, translate the forces into equation form and solve for the unknowns. Note that forces acting in opposite directions have opposite signs. By convention, forces acting downward or to the left are usually negative.

GRASP CHECK

If a problem has more than one system of interest, more than one free-body diagram is required to describe the external forces acting on the different systems.

a. True
b. False

WATCH PHYSICS

Newton's Third Law of Motion

This video explains Newton's third law of motion through examples involving push, normal force, and thrust (the force that propels a rocket or a jet).

Click to view content (https://www.openstax.org/l/astronaut)

GRASP CHECK

If the astronaut in the video wanted to move upward, in which direction should he throw the object? Why?

a. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., upward).
b. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., downward).
c. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., upward).
d. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., downward).

WORKED EXAMPLE

An Accelerating Subway Train

A physics teacher pushes a cart of demonstration equipment to a classroom, as in Figure 4.11. Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. To push the cart forward, the teacher's foot applies a force of 150 N in the opposite direction (backward) on the floor. Calculate the acceleration produced by the teacher. The force of friction, which opposes the motion, is 24.0 N.

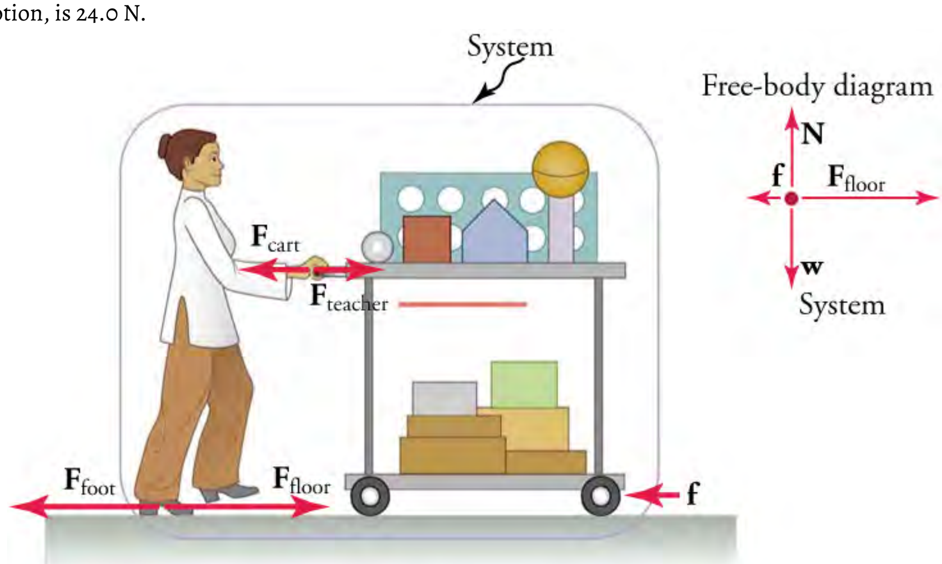

Figure 4.11

Strategy

Because they accelerate together, we define the system to be the teacher, the cart, and the equipment. The teacher pushes backward with a force $\mathbf{F}_{foot}$ of 150 N. According to Newton's third law, the floor exerts a forward force $\mathbf{F}_{floor}$ of 150 N on the system. Because all motion is horizontal, we can assume that no net force acts in the vertical direction, and the problem becomes one dimensional. As noted in the figure, the friction f opposes the motion and therefore acts opposite the direction of $\mathbf{F}_{floor}$.

We should not include the forces $\mathbf{F}_{\text{teacher}}$, $\mathbf{F}_{\text{cart}}$, or $\mathbf{F}_{\text{foot}}$ because these are exerted *by* the system, not *on* the system. We find the net external force by adding together the external forces acting on the system (see the free-body diagram in the figure) and then use Newton's second law to find the acceleration.

Solution

Newton's second law is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \tag{4.21}$$

The net external force on the system is the sum of the external forces: the force of the floor acting on the teacher, cart, and equipment (in the horizontal direction) and the force of friction. Because friction acts in the opposite direction, we assign it a negative value. Thus, for the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{F}_{\text{floor}} - \mathbf{f} = 150\text{ N} - 24.0\text{ N} = 126\text{ N}. \tag{4.22}$$

The mass of the system is the sum of the mass of the teacher, cart, and equipment.

$$m = (65.0 + 12.0 + 7.0)\text{ kg} = 84\text{ kg} \tag{4.23}$$

Insert these values of net F and m into Newton's second law to obtain the acceleration of the system.

$$\begin{aligned} \mathbf{a} &= \frac{\mathbf{F}_{\text{net}}}{m} \\ a &= \frac{126\text{ N}}{84\text{ kg}} = 1.5\text{ m/s}^2 \end{aligned} \tag{4.24}$$

$$F_1 < F_2 \tag{4.25}$$

Discussion

None of the forces between components of the system, such as between the teacher's hands and the cart, contribute to the net external force because they are internal to the system. Another way to look at this is to note that the forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the teacher on the cart is of equal magnitude but in the opposite direction of the force exerted by the cart on the teacher. In this case, both forces act on the same system, so they cancel. Defining the system was crucial to solving this problem.

Practice Problems

14. What is the equation for the normal force for a body with mass m that is at rest on a horizontal surface?

a. N = m
b. N = mg
c. N = mv
d. N = g

15. An object with mass m is at rest on the floor. What is the magnitude and direction of the normal force acting on it?

a. N = mv in upward direction
b. N = mg in upward direction
c. N = mv in downward direction
d. N = mg in downward direction

Check Your Understanding

16. What is Newton's third law of motion?

a. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude and acts in the direction of the applied force.
b. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude and acts in the direction of the applied force.
c. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude but acts in the direction opposite the direction of the applied force.
d. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude but

acts in the direction opposite the direction of the applied force.

17. Considering Newton's third law, why don't two equal and opposite forces cancel out each other?

a. Because the two forces act in the same direction
b. Because the two forces have different magnitudes
c. Because the two forces act on different systems
d. Because the two forces act in perpendicular directions