a. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a top spinning on its axis
b. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
c. Earth spinning on its own axis, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
d. Earth spinning on its own axis, blades of a working ceiling fan, and a top spinning on its own axis

8. What is the relative orientation of the radius and tangential velocity vectors of an object in uniform circular motion?
a. Tangential velocity vector is always parallel to the radius of the circular path along which the object moves.
b. Tangential velocity vector is always perpendicular to the radius of the circular path along which the object moves.
c. Tangential velocity vector is always at an acute angle to the radius of the circular path along which the object moves.
d. Tangential velocity vector is always at an obtuse angle to the radius of the circular path along which the object moves.

6.2 Uniform Circular Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe centripetal acceleration and relate it to linear acceleration
- Describe centripetal force and relate it to linear force
- Solve problems involving centripetal acceleration and centripetal force

Section Key Terms

centrifugal force	centripetal acceleration	centripetal force	uniform circular motion

Centripetal Acceleration

In the previous section, we defined circular motion. The simplest case of circular motion is **uniform circular motion**, where an object travels a circular path *at a constant speed*. Note that, unlike speed, the linear velocity of an object in circular motion is constantly changing because it is always changing direction. We know from kinematics that acceleration is a change in velocity, either in magnitude or in direction or both. Therefore, an object undergoing uniform circular motion is always accelerating, even though the magnitude of its velocity is constant.

You experience this acceleration yourself every time you ride in a car while it turns a corner. If you hold the steering wheel steady during the turn and move at a constant speed, you are executing uniform circular motion. What you notice is a feeling of sliding (or being flung, depending on the speed) away from the center of the turn. This isn't an actual force that is acting on you—it only happens because your body wants to continue moving in a straight line (as per Newton's first law) whereas the car is turning off this straight-line path. Inside the car it appears as if you are forced away from the center of the turn. This fictitious force is known as the **centrifugal force**. The sharper the curve and the greater your speed, the more noticeable this effect becomes.

Figure 6.7 shows an object moving in a circular path at constant speed. The direction of the instantaneous tangential velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity; in this case it points roughly toward the center of rotation. (The center of rotation is at the center of the circular path). If we imagine Δs becoming smaller and smaller, then the acceleration would point *exactly* toward the center of rotation, but this case is hard to draw. We call the acceleration of an object moving in uniform circular motion the **centripetal acceleration** $\mathbf{a}_c$ because centripetal means *center seeking*.

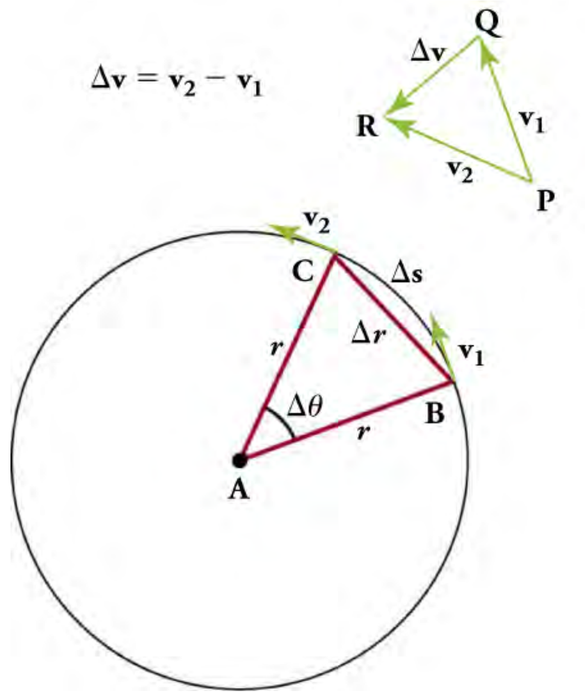


Figure 6.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ is seen to point approximately toward the center of curvature (see small inset). For an extremely small value of Δs, $\Delta \mathbf{v}$ points exactly toward the center of the circle (but this is hard to draw). Because $\mathbf{a}_c = \Delta \mathbf{v}/\Delta t$, the acceleration is also toward the center, so $\mathbf{a}_c$ is called centripetal acceleration.

Now that we know that the direction of centripetal acceleration is toward the center of rotation, let's discuss the magnitude of centripetal acceleration. For an object traveling at speed v in a circular path with radius r, the magnitude of centripetal acceleration is

$$\mathbf{a}_c = \frac{v^2}{r}.$$

Centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you may have noticed when driving a car, because the car actually pushes you toward the center of the turn. But it is a bit surprising that $\mathbf{a}_c$ is proportional to the speed squared. This means, for example, that the acceleration is four times greater when you take a curve at 100 km/h than at 50 km/h.

We can also express $\mathbf{a}_c$ in terms of the magnitude of angular velocity. Substituting $v = r\omega$ into the equation above, we get $a_c = \frac{(r\omega)^2}{r} = r\omega^2$. Therefore, the magnitude of centripetal acceleration in terms of the magnitude of angular velocity is

$$\mathbf{a}_c = r\omega^2. \tag{6.9}$$

TIPS FOR SUCCESS

The equation expressed in the form $a_c = r\omega^2$ is useful for solving problems where you know the angular velocity rather than the tangential velocity.

Virtual Physics

Ladybug Motion in 2D

In this simulation, you experiment with the position, velocity, and acceleration of a ladybug in circular and elliptical motion. Switch the type of motion from linear to circular and observe the velocity and acceleration vectors. Next, try elliptical motion and notice how the velocity and acceleration vectors differ from those in circular motion.

Click to view content (https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/)

GRASP CHECK

In uniform circular motion, what is the angle between the acceleration and the velocity? What type of acceleration does a body experience in the uniform circular motion?

a. The angle between acceleration and velocity is 0°, and the body experiences linear acceleration.
b. The angle between acceleration and velocity is 0°, and the body experiences centripetal acceleration.
c. The angle between acceleration and velocity is 90°, and the body experiences linear acceleration.
d. The angle between acceleration and velocity is 90°, and the body experiences centripetal acceleration.

Centripetal Force

Because an object in uniform circular motion undergoes constant acceleration (by changing direction), we know from Newton's second law of motion that there must be a constant net external force acting on the object.

Any force or combination of forces can cause a centripetal acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, the friction between a road and the tires of a car as it goes around a curve, or the normal force of a roller coaster track on the cart during a loop-the-loop.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of rotation, the same as for centripetal acceleration. According to Newton's second law of motion, a net force causes the acceleration of mass according to $\mathbf{F}_{\text{net}} = m\mathbf{a}$. For uniform circular motion, the acceleration is centripetal acceleration: $\mathbf{a} = \mathbf{a}_\text{c}$. Therefore, the magnitude of centripetal force, $\mathbf{F}_\text{c}$, is $\mathbf{F}_\text{c} = m\mathbf{a}_\text{c}$.

By using the two different forms of the equation for the magnitude of centripetal acceleration, $\mathbf{a}_\text{c} = v^2/r$ and $\mathbf{a}_c = r\omega^2$, we get two expressions involving the magnitude of the centripetal force $\mathbf{F}_\text{c}$. The first expression is in terms of tangential speed, the second is in terms of angular speed: $\mathbf{F}_\text{c} = m\frac{v^2}{r}$ and $\mathbf{F}_\text{c} = mr\omega^2$.

Both forms of the equation depend on mass, velocity, and the radius of the circular path. You may use whichever expression for centripetal force is more convenient. Newton's second law also states that the object will accelerate in the same direction as the net force. By definition, the centripetal force is directed towards the center of rotation, so the object will also accelerate towards the center. A straight line drawn from the circular path to the center of the circle will always be perpendicular to the tangential velocity. Note that, if you solve the first expression for r, you get

$$r = \frac{mv^2}{\mathbf{F}_\text{c}}.$$

From this expression, we see that, for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

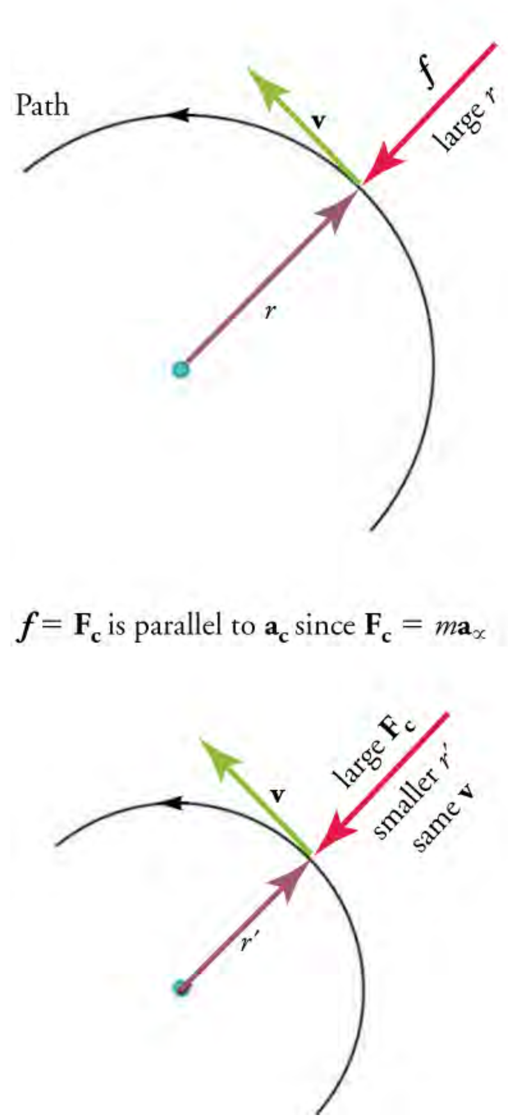


Figure 6.8 In this figure, the frictional force f serves as the centripetal force $\mathbf{F}_c$. Centripetal force is perpendicular to tangential velocity and causes uniform circular motion. The larger the centripetal force $\mathbf{F}_c$, the smaller is the radius of curvature r and the sharper is the curve. The lower curve has the same velocity $\mathbf{v}$, but a larger centripetal force $\mathbf{F}_c$ produces a smaller radius r' .

WATCH PHYSICS

Centripetal Force and Acceleration Intuition

This video explains why a centripetal force creates centripetal acceleration and uniform circular motion. It also covers the difference between speed and velocity and shows examples of uniform circular motion.

Click to view content (https://www.youtube.com/embed/vZOk8NnjILg)

GRASP CHECK

Imagine that you are swinging a yoyo in a vertical clockwise circle in front of you, perpendicular to the direction you are facing. Now, imagine that the string breaks just as the yoyo reaches its bottommost position, nearest the floor. Which of the following describes the path of the yoyo after the string breaks?

a. The yoyo will fly upward in the direction of the centripetal force.
b. The yoyo will fly downward in the direction of the centripetal force.

c. The yoyo will fly to the left in the direction of the tangential velocity.
d. The yoyo will fly to the right in the direction of the tangential velocity.

Solving Centripetal Acceleration and Centripetal Force Problems

To get a feel for the typical magnitudes of centripetal acceleration, we'll do a lab estimating the centripetal acceleration of a tennis racket and then, in our first Worked Example, compare the centripetal acceleration of a car rounding a curve to gravitational acceleration. For the second Worked Example, we'll calculate the force required to make a car round a curve.

Snap Lab

Estimating Centripetal Acceleration

In this activity, you will measure the swing of a golf club or tennis racket to estimate the centripetal acceleration of the end of the club or racket. You may choose to do this in slow motion. Recall that the equation for centripetal acceleration is $\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$.

- One tennis racket or golf club
- One timer
- One ruler or tape measure

Procedure

1. Work with a partner. Stand a safe distance away from your partner as he or she swings the golf club or tennis racket.
2. Describe the motion of the swing—is this uniform circular motion? Why or why not?
3. Try to get the swing as close to uniform circular motion as possible. What adjustments did your partner need to make?
4. Measure the radius of curvature. What did you physically measure?
5. By using the timer, find either the linear or angular velocity, depending on which equation you decide to use.
6. What is the approximate centripetal acceleration based on these measurements? How accurate do you think they are? Why? How might you and your partner make these measurements more accurate?

GRASP CHECK

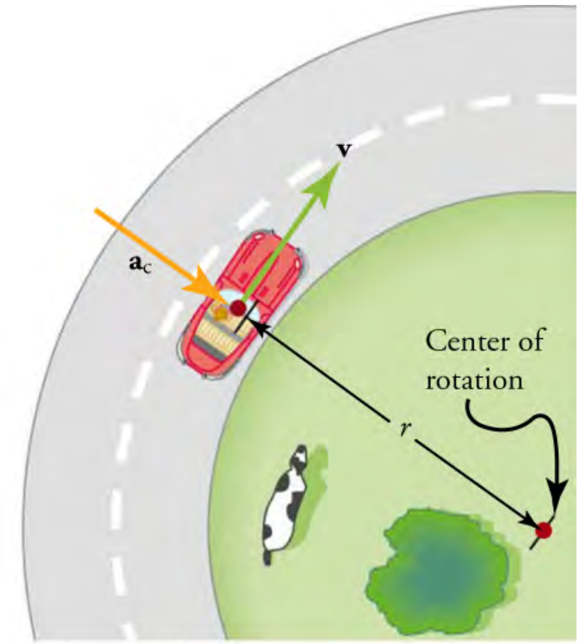
Was it more useful to use the equation $a_c = \frac{v^2}{r}$ or $a_c = r\omega^2$ in this activity? Why?

a. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be easier.
b. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be easier.
c. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be difficult.
d. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be difficult.

WORKED EXAMPLE

Comparing Centripetal Acceleration of a Car Rounding a Curve with Acceleration Due to Gravity

A car follows a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h). What is the magnitude of the car's centripetal acceleration? Compare the centripetal acceleration for this fairly gentle curve taken at highway speed with acceleration due to gravity (*g*).



Car around corner

Strategy

Because linear rather than angular speed is given, it is most convenient to use the expression $\mathbf{a}_c = \frac{v^2}{r}$ to find the magnitude of the centripetal acceleration.

Solution

Entering the given values of $v = 25.0$ m/s and $r = 500$ m into the expression for $\mathbf{a}_c$ gives

$$\begin{aligned}\mathbf{a}_c &= \frac{v^2}{r}\\ &= \frac{(25.0\text{ m/s})^2}{500\text{ m}}\\ &= 1.25\text{m/s}^2.\end{aligned}$$

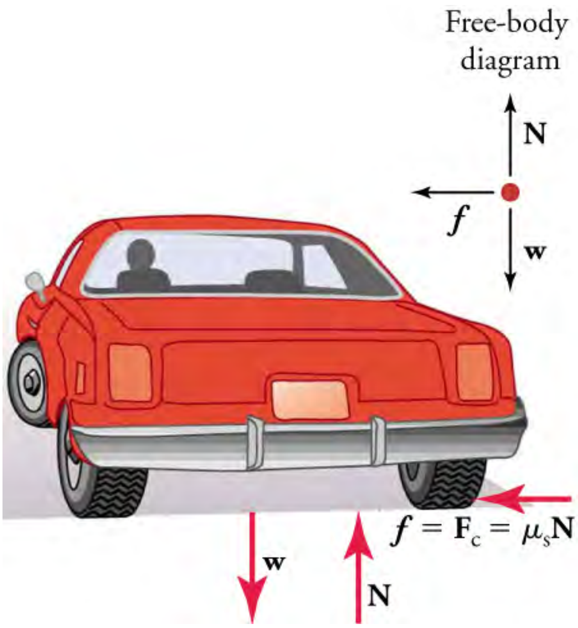
Discussion

To compare this with the acceleration due to gravity ($g = 9.80$ m/s^2), we take the ratio $\mathbf{a}_c/g = (1.25\text{ m/s}^2)/(9.80\text{m/s}^2) = 0.128$. Therefore, $\mathbf{a}_c = 0.128g$, which means that the centripetal acceleration is about one tenth the acceleration due to gravity.

WORKED EXAMPLE

Frictional Force on Car Tires Rounding a Curve

a. Calculate the centripetal force exerted on a 900 kg car that rounds a 600-m-radius curve on horizontal ground at 25.0 m/s.
b. Static friction prevents the car from slipping. Find the magnitude of the frictional force between the tires and the road that allows the car to round the curve without sliding off in a straight line.



Strategy and Solution for (a)

We know that $\mathbf{F}_c = m\frac{v^2}{r}$. Therefore,

$$\begin{aligned}\mathbf{F}_c &= m\frac{v^2}{r}\\ &= \frac{(900\ \text{kg})(25.0\ \text{m/s})^2}{600\ \text{m}}\\ &= 938\ \text{N}.\end{aligned}$$

Strategy and Solution for (b)

The image above shows the forces acting on the car while rounding the curve. In this diagram, the car is traveling into the page as shown and is turning to the left. Friction acts toward the left, accelerating the car toward the center of the curve. Because friction is the only horizontal force acting on the car, it provides all of the centripetal force in this case. Therefore, the force of friction is the centripetal force in this situation and points toward the center of the curve.

$$f = \mathbf{F}_c = 938\ \text{N}$$

Discussion

Since we found the force of friction in part (b), we could also solve for the coefficient of friction, since $f = \mu_s \text{N} = \mu_s mg$.

Practice Problems

9. What is the centripetal acceleration of an object with speed $12\ \text{m/s}$ going along a path of radius $2.0\ \text{m}$?
   a. $6\ \text{m/s}$
   b. $72\ \text{m/s}$
   c. $6\ \text{m/s}^2$
   d. $72\ \text{m/s}^2$

10. Calculate the centripetal acceleration of an object following a path with a radius of a curvature of 0.2 m and at an angular velocity of 5 rad/s.
    a. 1 m/s
    b. 5 m/s
    c. $1\ \text{m/s}^2$
    d. $5\ \text{m/s}^2$

Check Your Understanding

11. What is uniform circular motion?

a. Uniform circular motion is when an object accelerates on a circular path at a constantly increasing velocity.
b. Uniform circular motion is when an object travels on a circular path at a variable acceleration.
c. Uniform circular motion is when an object travels on a circular path at a constant speed.
d. Uniform circular motion is when an object travels on a circular path at a variable speed.

12. What is centripetal acceleration?
a. The acceleration of an object moving in a circular path and directed radially toward the center of the circular orbit
b. The acceleration of an object moving in a circular path and directed tangentially along the circular path
c. The acceleration of an object moving in a linear path and directed in the direction of motion of the object
d. The acceleration of an object moving in a linear path and directed in the direction opposite to the motion of the object

13. Is there a net force acting on an object in uniform circular motion?
a. Yes, the object is accelerating, so a net force must be acting on it.
b. Yes, because there is no acceleration.
c. No, because there is acceleration.
d. No, because there is no acceleration.

14. Identify two examples of forces that can cause centripetal acceleration.
a. The force of Earth's gravity on the moon and the normal force
b. The force of Earth's gravity on the moon and the tension in the rope on an orbiting tetherball
c. The normal force and the force of friction acting on a moving car
d. The normal force and the tension in the rope on a tetherball

6.3 Rotational Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe rotational kinematic variables and equations and relate them to their linear counterparts
- Describe torque and lever arm
- Solve problems involving torque and rotational kinematics

Section Key Terms

angular acceleration	kinematics of rotational motion	lever arm
tangential acceleration	torque	

Rotational Kinematics

In the section on uniform circular motion, we discussed motion in a circle at constant speed and, therefore, constant angular velocity. However, there are times when angular velocity is not constant—rotational motion can speed up, slow down, or reverse directions. Angular velocity is not constant when a spinning skater pulls in her arms, when a child pushes a merry-go-round to make it rotate, or when a CD slows to a halt when switched off. In all these cases, **angular acceleration** occurs because the angular velocity $\boldsymbol{\omega}$ changes. The faster the change occurs, the greater is the angular acceleration. Angular acceleration $\boldsymbol{\alpha}$ is the rate of change of angular velocity. In equation form, angular acceleration is

$$\boldsymbol{\alpha} = \frac{\Delta \boldsymbol{\omega}}{\Delta t},$$

where $\Delta \boldsymbol{\omega}$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If $\boldsymbol{\omega}$ increases, then $\boldsymbol{\alpha}$ is positive. If $\boldsymbol{\omega}$ decreases, then $\boldsymbol{\alpha}$ is negative. Keep in mind that, by convention, counterclockwise is the positive direction and clockwise is the negative direction. For example, the skater in Figure 6.9 is rotating counterclockwise as seen from above, so her angular velocity is positive. Acceleration would be negative, for example, when an object that is rotating counterclockwise slows down. It would be positive when an object that is rotating counterclockwise speeds up.