a. Uniform circular motion is when an object accelerates on a circular path at a constantly increasing velocity.
b. Uniform circular motion is when an object travels on a circular path at a variable acceleration.
c. Uniform circular motion is when an object travels on a circular path at a constant speed.
d. Uniform circular motion is when an object travels on a circular path at a variable speed.

12. What is centripetal acceleration?
a. The acceleration of an object moving in a circular path and directed radially toward the center of the circular orbit
b. The acceleration of an object moving in a circular path and directed tangentially along the circular path
c. The acceleration of an object moving in a linear path and directed in the direction of motion of the object
d. The acceleration of an object moving in a linear path and directed in the direction opposite to the motion of the object

13. Is there a net force acting on an object in uniform circular motion?
a. Yes, the object is accelerating, so a net force must be acting on it.
b. Yes, because there is no acceleration.
c. No, because there is acceleration.
d. No, because there is no acceleration.

14. Identify two examples of forces that can cause centripetal acceleration.
a. The force of Earth's gravity on the moon and the normal force
b. The force of Earth's gravity on the moon and the tension in the rope on an orbiting tetherball
c. The normal force and the force of friction acting on a moving car
d. The normal force and the tension in the rope on a tetherball

6.3 Rotational Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe rotational kinematic variables and equations and relate them to their linear counterparts
- Describe torque and lever arm
- Solve problems involving torque and rotational kinematics

Section Key Terms

angular acceleration	kinematics of rotational motion	lever arm
tangential acceleration	torque	

Rotational Kinematics

In the section on uniform circular motion, we discussed motion in a circle at constant speed and, therefore, constant angular velocity. However, there are times when angular velocity is not constant—rotational motion can speed up, slow down, or reverse directions. Angular velocity is not constant when a spinning skater pulls in her arms, when a child pushes a merry-go-round to make it rotate, or when a CD slows to a halt when switched off. In all these cases, **angular acceleration** occurs because the angular velocity $\boldsymbol{\omega}$ changes. The faster the change occurs, the greater is the angular acceleration. Angular acceleration $\boldsymbol{\alpha}$ is the rate of change of angular velocity. In equation form, angular acceleration is

$$\boldsymbol{\alpha} = \frac{\Delta \boldsymbol{\omega}}{\Delta t},$$

where $\Delta\boldsymbol{\omega}$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If $\boldsymbol{\omega}$ increases, then $\boldsymbol{\alpha}$ is positive. If $\boldsymbol{\omega}$ decreases, then $\boldsymbol{\alpha}$ is negative. Keep in mind that, by convention, counterclockwise is the positive direction and clockwise is the negative direction. For example, the skater in Figure 6.9 is rotating counterclockwise as seen from above, so her angular velocity is positive. Acceleration would be negative, for example, when an object that is rotating counterclockwise slows down. It would be positive when an object that is rotating counterclockwise speeds up.

Figure 6.9 A figure skater spins in the counterclockwise direction, so her angular velocity is normally considered to be positive. (Luu, Wikimedia Commons)

The relationship between the magnitudes of **tangential acceleration**, **a**, and angular acceleration,

$$\boldsymbol{\alpha}\text{, is}\mathbf{a} = r\boldsymbol{\alpha}\text{or}\boldsymbol{\alpha} = \frac{\mathbf{a}}{r}. \tag{6.10}$$

These equations mean that the magnitudes of tangential acceleration and angular acceleration are directly proportional to each other. The greater the angular acceleration, the larger the change in tangential acceleration, and vice versa. For example, consider riders in their pods on a Ferris wheel at rest. A Ferris wheel with greater angular acceleration will give the riders greater tangential acceleration because, as the Ferris wheel increases its rate of spinning, it also increases its tangential velocity. Note that the radius of the spinning object also matters. For example, for a given angular acceleration $\boldsymbol{\alpha}$, a smaller Ferris wheel leads to a smaller tangential acceleration for the riders.

TIPS FOR SUCCESS

Tangential acceleration is sometimes denoted $\mathbf{a}_t$. It is a linear acceleration in a direction tangent to the circle at the point of interest in circular or rotational motion. Remember that tangential acceleration is parallel to the tangential velocity (either in the same direction or in the opposite direction.) Centripetal acceleration is always perpendicular to the tangential velocity.

So far, we have defined three rotational variables: θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$. These are the angular versions of the linear variables x, $\mathbf{v}$, and $\mathbf{a}$. Table 6.2 shows how they are related.

Rotational	Linear	Relationship
θ	x	$\theta = \frac{x}{r}$

Table 6.2 Rotational and Linear Variables

Rotational	Linear	Relationship
$\boldsymbol{\omega}$	$\mathbf{v}$	$\boldsymbol{\omega} = \frac{\mathbf{v}}{r}$
$\boldsymbol{\alpha}$	$\mathbf{a}$	$\boldsymbol{\alpha} = \frac{\mathbf{a}}{r}$

Table 6.2 Rotational and Linear Variables

We can now begin to see how rotational quantities like θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$ are related to each other. For example, if a motorcycle wheel that starts at rest has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. Putting this in terms of the variables, if the wheel's angular acceleration $\boldsymbol{\alpha}$ is large for a long period of time t, then the final angular velocity $\boldsymbol{\omega}$ and angle of rotation θ are large. In the case of linear motion, if an object starts at rest and undergoes a large linear acceleration, then it has a large final velocity and will have traveled a large distance.

The **kinematics of rotational motion** describes the relationships between the angle of rotation, angular velocity, angular acceleration, and time. It only *describes* motion—it does not include any forces or masses that may affect rotation (these are part of dynamics). Recall the kinematics equation for linear motion: $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ (constant **a**).

As in linear kinematics, we assume **a** is constant, which means that angular acceleration $\boldsymbol{\alpha}$ is also a constant, because $\mathbf{a} = r\boldsymbol{\alpha}$. The equation for the kinematics relationship between $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, and t is

$$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t(\text{constant}\alpha),$$

where $\boldsymbol{\omega}_0$ is the initial angular velocity. Notice that the equation is identical to the linear version, except with angular analogs of the linear variables. In fact, all of the linear kinematics equations have rotational analogs, which are given in Table 6.3. These equations can be used to solve rotational or linear kinematics problem in which **a** and $\boldsymbol{\alpha}$ are constant.

Rotational	Linear	
$\theta = \overline{\boldsymbol{\omega}}t$	$x = \overline{\mathbf{v}}t$	
$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$	$\mathbf{v} = \mathbf{v}_0 + \boldsymbol{\alpha}t$	constant $\boldsymbol{\alpha}$, **a**
$\theta = \boldsymbol{\omega}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$	$x = \mathbf{v}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$	constant $\boldsymbol{\alpha}$, **a**
$\boldsymbol{\omega}^2 = \boldsymbol{\omega}_0{}^2 + 2\boldsymbol{\alpha}\theta$	$\mathbf{v}^2 = \mathbf{v}_0{}^2 + 2\boldsymbol{\alpha}x$	constant $\boldsymbol{\alpha}$, **a**

Table 6.3 Equations for Rotational Kinematics

In these equations, $\boldsymbol{\omega}_0$ and $\mathbf{v}_0$ are initial values, t_0 is zero, and the average angular velocity $\overline{\boldsymbol{\omega}}$ and average velocity $\overline{\mathbf{v}}$ are

$$\overline{\boldsymbol{\omega}} = \frac{\boldsymbol{\omega}_0 + \boldsymbol{\omega}}{2} \text{and} \overline{\mathbf{v}} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}. \tag{6.11}$$

FUN IN PHYSICS

Storm Chasing

Figure 6.10 Tornadoes descend from clouds in funnel-like shapes that spin violently. (Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Storm chasers tend to fall into one of three groups: Amateurs chasing tornadoes as a hobby, atmospheric scientists gathering data for research, weather watchers for news media, or scientists having fun under the guise of work. Storm chasing is a dangerous pastime because tornadoes can change course rapidly with little warning. Since storm chasers follow in the wake of the destruction left by tornadoes, changing flat tires due to debris left on the highway is common. The most active part of the world for tornadoes, called *tornado alley*, is in the central United States, between the Rocky Mountains and Appalachian Mountains.

Tornadoes are perfect examples of rotational motion in action in nature. They come out of severe thunderstorms called supercells, which have a column of air rotating around a horizontal axis, usually about four miles across. The difference in wind speeds between the strong cold winds higher up in the atmosphere in the jet stream and weaker winds traveling north from the Gulf of Mexico causes the column of rotating air to shift so that it spins around a vertical axis, creating a tornado.

Tornadoes produce wind speeds as high as 500 km/h (approximately 300 miles/h), particularly at the bottom where the funnel is narrowest because the rate of rotation increases as the radius decreases. They blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw.

GRASP CHECK

What is the physics term for the eye of the storm? Why would winds be weaker at the eye of the tornado than at its outermost edge?

a. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to radius of curvature.
b. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to radius of curvature.
c. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to the square of the radius of curvature.
d. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to the square of the radius of curvature.

Torque

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity. The farther the force is applied from the pivot point (or fulcrum), the greater the angular acceleration. For example, a door opens slowly if you push too close to its hinge, but opens easily if you push far from the hinges. Furthermore, we know that the more massive the door is, the more slowly it opens; this is because angular acceleration is inversely proportional to mass. These relationships are very similar to the relationships between force, mass, and acceleration from Newton's second law of motion. Since we have already covered the angular versions of distance, velocity and time, you may wonder what the angular version of force is, and how it relates to linear force.

The angular version of force is **torque** $\boldsymbol{\tau}$, which is the turning effectiveness of a force. See Figure 6.11. The equation for the magnitude of torque is

$$\boldsymbol{\tau} = r\mathbf{F}\sin\theta,$$

where r is the magnitude of the **lever arm**, $\mathbf{F}$ is the magnitude of the linear force, and θ is the angle between the lever arm and the force. The lever arm is the vector from the point of rotation (pivot point or fulcrum) to the location where force is applied. Since the magnitude of the lever arm is a distance, its units are in meters, and torque has units of N·m. Torque is a vector quantity and has the same direction as the angular acceleration that it produces.

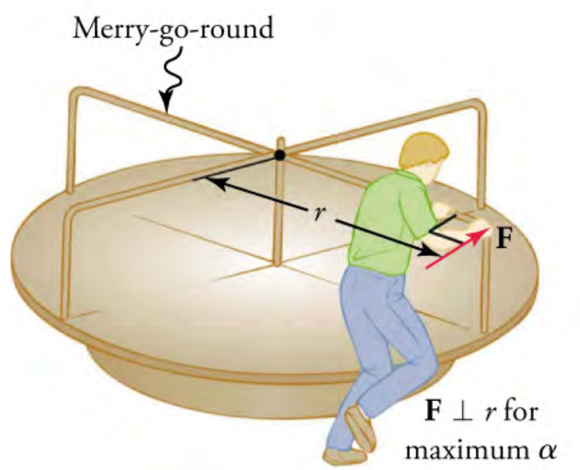


Figure 6.11 A man pushes a merry-go-round at its edge and perpendicular to the lever arm to achieve maximum torque.

Applying a stronger torque will produce a greater angular acceleration. For example, the harder the man pushes the merry-go-round in Figure 6.11, the faster it accelerates. Furthermore, the more massive the merry-go-round is, the slower it accelerates for the same torque. If the man wants to maximize the effect of his force on the merry-go-round, he should push as far from the center as possible to get the largest lever arm and, therefore, the greatest torque and angular acceleration. Torque is also maximized when the force is applied perpendicular to the lever arm.

Solving Rotational Kinematics and Torque Problems

Just as linear forces can balance to produce zero net force and no linear acceleration, the same is true of rotational motion. When two torques of equal magnitude act in opposing directions, there is no net torque and no angular acceleration, as you can see in the following video. If zero net torque acts on a system spinning at a constant angular velocity, the system will continue to spin at the same angular velocity.

WATCH PHYSICS

Introduction to Torque

This video (https://www.khanacademy.org/science/physics/torque-angular-momentum/torque-tutorial/v/introduction-to-torque) defines torque in terms of moment arm (which is the same as lever arm). It also covers a problem with forces acting in opposing directions about a pivot point. (At this stage, you can ignore Sal's references to work and mechanical advantage.)

GRASP CHECK

Click to view content (https://www.openstax.org/l/28torque)

If the net torque acting on the ruler from the example was positive instead of zero, what would this say about the angular acceleration? What would happen to the ruler over time?

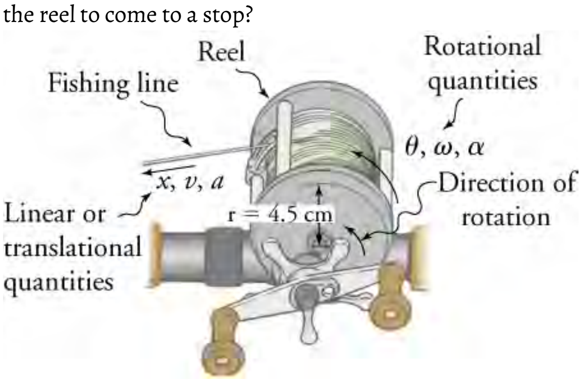
a. The ruler is in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.
b. The ruler is not in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.
c. The ruler is not in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.
d. The ruler is in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.

Now let's look at examples applying rotational kinematics to a fishing reel and the concept of torque to a merry-go-round.

WORKED EXAMPLE

Calculating the Time for a Fishing Reel to Stop Spinning

A deep-sea fisherman uses a fishing rod with a reel of radius 4.50 cm. A big fish takes the bait and swims away from the boat, pulling the fishing line from his fishing reel. As the fishing line unwinds from the reel, the reel spins at an angular velocity of 220 rad/s. The fisherman applies a brake to the spinning reel, creating an angular acceleration of –300 rad/s². How long does it take the reel to come to a stop?



Strategy

We are asked to find the time t for the reel to come to a stop. The magnitude of the initial angular velocity is $\boldsymbol{\omega}_0 = 220$ rad/s, and the magnitude of the final angular velocity $\boldsymbol{\omega} = 0$. The signed magnitude of the angular acceleration is $\boldsymbol{\alpha} = -300$ rad/s², where the minus sign indicates that it acts in the direction opposite to the angular velocity. Looking at the rotational kinematic equations, we see all quantities but t are known in the equation $\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$, making it the easiest equation to use for this problem.

Solution

The equation to use is $\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$.

We solve the equation algebraically for t, and then insert the known values.

$$\begin{aligned} t &= \frac{\omega - \omega_0}{\alpha} \\ &= \frac{0 - 220\ \text{rad/s}}{-300 \text{rad/s}^2} \\ &= 0.733\ \text{s} \end{aligned} \tag{6.12}$$

Discussion

The time to stop the reel is fairly small because the acceleration is fairly large. Fishing lines sometimes snap because of the forces involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration and therefore a smaller force.

WORKED EXAMPLE

Calculating the Torque on a Merry-Go-Round

Consider the man pushing the playground merry-go-round in Figure 6.11. He exerts a force of 250 N at the edge of the merry-go-round and perpendicular to the radius, which is 1.50 m. How much torque does he produce? Assume that friction acting on the merry-go-round is negligible.

Strategy

To find the torque, note that the applied force is perpendicular to the radius and that friction is negligible.

Solution

$$\begin{aligned}\tau &= r\mathbf{F}\sin\theta \\ &= (1.50\text{ m})(250\text{ N})\sin\left(\frac{\pi}{2}\right). \\ &= 375\text{ N}\cdot\text{m}\end{aligned} \tag{6.13}$$

Discussion

The man maximizes the torque by applying force perpendicular to the lever arm, so that $\theta = \frac{\pi}{2}$ and $\sin\theta = 1$. The man also maximizes his torque by pushing at the outer edge of the merry-go-round, so that he gets the largest-possible lever arm.

Practice Problems

15. How much torque does a person produce if he applies a 12 N force 1.0 m away from the pivot point, perpendicularly to the lever arm?
    a. $\frac{1}{144}\text{ N-m}$
    b. $\frac{1}{12}\text{ N-m}$
    c. 12 N-m
    d. 144 N-m

16. An object's angular velocity changes from 3 rad/s clockwise to 8 rad/s clockwise in 5 s. What is its angular acceleration?
    a. 0.6 rad/s^2
    b. 1.6 rad/s^2
    c. 1 rad/s^2
    d. 5 rad/s^2

Check Your Understanding

17. What is angular acceleration?
    a. Angular acceleration is the rate of change of the angular displacement.
    b. Angular acceleration is the rate of change of the angular velocity.
    c. Angular acceleration is the rate of change of the linear displacement.
    d. Angular acceleration is the rate of change of the linear velocity.

18. What is the equation for angular acceleration, α? Assume θ is the angle, ω is the angular velocity, and t is time.
    a. $\alpha = \frac{\Delta\omega}{\Delta t}$
    b. $\alpha = \Delta\omega\Delta t$
    c. $\alpha = \frac{\Delta\theta}{\Delta t}$
    d. $\alpha = \Delta\theta\Delta t$

19. Which of the following best describes torque?
    a. It is the rotational equivalent of a force.
    b. It is the force that affects linear motion.
    c. It is the rotational equivalent of acceleration.
    d. It is the acceleration that affects linear motion.

20. What is the equation for torque?

a. $\tau = F\,cos\theta\,r$
b. $\tau = \frac{F \sin\theta}{r}$
c. $\tau = rF\cos\theta$
d. $\tau = rF\sin\theta$