KEY TERMS

angle of rotation the ratio of the arc length to the radius of curvature of a circular path

angular acceleration the rate of change of angular velocity with time

angular velocity ($\boldsymbol{\omega}$) the rate of change in the angular position of an object following a circular path

arc length (Δs) the distance traveled by an object along a circular path

centrifugal force a fictitious force that acts in the direction opposite the centripetal acceleration

centripetal acceleration the acceleration of an object moving in a circle, directed toward the center of the circle

centripetal force any force causing uniform circular motion

circular motion the motion of an object along a circular path

kinematics of rotational motion the relationships between rotation angle, angular velocity, angular acceleration, and time

lever arm the distance between the point of rotation (pivot point) and the location where force is applied

radius of curvature the distance between the center of a circular path and the path

rotational motion the circular motion of an object about an axis of rotation

spin rotation about an axis that goes through the center of mass of the object

tangential acceleration the acceleration in a direction tangent to the circular path of motion and in the same direction or opposite direction as the tangential velocity

tangential velocity the instantaneous linear velocity of an object in circular or rotational motion

torque the effectiveness of a force to change the rotational speed of an object

uniform circular motion the motion of an object in a circular path at constant speed

SECTION SUMMARY

6.1 Angle of Rotation and Angular Velocity

- Circular motion is motion in a circular path.
- The angle of rotation $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature.
- The arc length Δs is the distance traveled along a circular path and r is the radius of curvature of the circular path.
- The angle of rotation $\Delta\theta$ is measured in units of radians (rad), where $2\ \pi\text{rad} = 360^\circ = 1$ revolution.
- Angular velocity $\boldsymbol{\omega}$ is the rate of change of an angle, where a rotation $\Delta\theta$ occurs in a time Δt .
- The units of angular velocity are radians per second (rad/s).
- Tangential speed v and angular speed ω are related by $v = r\omega$, and tangential velocity has units of m/s.
- The direction of angular velocity is along the axis of rotation, toward (away) from you for clockwise (counterclockwise) motion.

6.2 Uniform Circular Motion

- Centripetal acceleration $\mathbf{a}_c$ is the acceleration experienced while in uniform circular motion.
- Centripetal acceleration force is a *center-seeking* force that always points toward the center of rotation, perpendicular to the linear velocity, in the same direction as the net force, and in the direction opposite that of the radius vector.
- The standard unit for centripetal acceleration is m/s^2.
- Centripetal force $\mathbf{F}_c$ is any net force causing uniform circular motion.

6.3 Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
- Torque is the effectiveness of a force to change the rotational speed of an object. Torque is the rotational analog of force.
- The lever arm is the distance between the point of rotation (pivot point) and the location where force is applied.
- Torque is maximized by applying force perpendicular to the lever arm and at a point as far as possible from the pivot point or fulcrum. If torque is zero, angular acceleration is zero.

KEY EQUATIONS

6.1 Angle of Rotation and Angular Velocity

Angle of rotation	$\Delta\theta = \frac{\Delta s}{r}$
Angular speed:	$\omega = \frac{\Delta\theta}{\Delta t}$
Tangential speed:	$v = r\omega$

6.2 Uniform Circular Motion

Centripetal acceleration	$\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$
Centripetal force	$\mathbf{F}_c = m\mathbf{a}_c, \mathbf{F}_c = m\frac{v^2}{r}, \mathbf{F}_c = mr\omega^2$

6.3 Rotational Motion

Angular acceleration	$\boldsymbol{\alpha} = \frac{\Delta\boldsymbol{\omega}}{\Delta t}$
Rotational kinematic equations	$\theta = \boldsymbol{\omega} t, \boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha} t, \theta = \boldsymbol{\omega}_0 t + \frac{1}{2}\boldsymbol{\alpha} t^2, \boldsymbol{\omega}^2 = \boldsymbol{\omega}_0{}^2 + 2\boldsymbol{\alpha}\theta$
Tangential (linear) acceleration	$\mathbf{a} = r\boldsymbol{\alpha}$
Torque	$\boldsymbol{\tau} = r\mathbf{F}\sin\theta$

CHAPTER REVIEW

Concept Items

6.1 Angle of Rotation and Angular Velocity

1. One revolution is equal to how many radians? Degrees?
   a. $1\text{ rev} = \pi\text{ rad} = 180^\circ$
   b. $1\text{ rev} = \pi\text{ rad} = 360^\circ$
   c. $1\text{ rev} = 2\pi\text{ rad} = 180^\circ$
   d. $1\text{ rev} = 2\pi\text{ rad} = 360^\circ$

2. What is tangential velocity?
   a. Tangential velocity is the average linear velocity of an object in a circular motion.
   b. Tangential velocity is the instantaneous linear velocity of an object undergoing rotational motion.
   c. Tangential velocity is the average angular velocity of an object in a circular motion.
   d. Tangential velocity is the instantaneous angular velocity of an object in a circular motion.

3. What kind of motion is called *spin*?
   a. Spin is rotational motion of an object about an axis parallel to the axis of the object.
   b. Spin is translational motion of an object about an axis parallel to the axis of the object.
   c. Spin is the rotational motion of an object about its center of mass.
   d. Spin is translational motion of an object about its own axis.

6.2 Uniform Circular Motion

4. What is the equation for centripetal acceleration in terms of angular velocity and the radius?
   a. $a_c = \frac{\omega^2}{r}$
   b. $a_c = \frac{\omega}{r}$
   c. $a_c = r\omega^2$
   d. $a_c = r\omega$

5. How can you express centripetal force in terms of centripetal acceleration?
   a. $F_c = \frac{a_c^2}{m}$
   b. $F_c = \frac{a_c}{m}$
   c. $F_c = ma_c^2$
   d. $F_c = ma_c$

6. What is meant by the word centripetal?
   a. center-seeking
   b. center-avoiding
   c. central force
   d. central acceleration

6.3 Rotational Motion

7. Conventionally, for which direction of rotation of an object is angular acceleration considered positive?
   a. the positive *x* direction of the coordinate system
   b. the negative *x* direction of the coordinate system
   c. the counterclockwise direction
   d. the clockwise direction

8. When you push a door closer to the hinges, why does it open more slowly?
   a. It opens slowly, because the lever arm is shorter so the torque is large.
   b. It opens slowly because the lever arm is longer so the torque is large.
   c. It opens slowly, because the lever arm is shorter so the torque is less.
   d. It opens slowly, because the lever arm is longer so the torque is less.

9. When is angular acceleration negative?
   a. Angular acceleration is the rate of change of the displacement and is negative when ω increases.
   b. Angular acceleration is the rate of change of the displacement and is negative when ω decreases.
   c. Angular acceleration is the rate of change of angular velocity and is negative when ω increases.
   d. Angular acceleration is the rate of change of angular velocity and is negative when ω decreases.

Critical Thinking Items

6.1 Angle of Rotation and Angular Velocity

10. When the radius of the circular path of rotational motion increases, what happens to the arc length for a given angle of rotation?
   a. The arc length is directly proportional to the radius of the circular path, and it increases with the radius.
   b. The arc length is inversely proportional to the radius of the circular path, and it decreases with the radius.
   c. The arc length is directly proportional to the radius of the circular path, and it decreases with the radius.
   d. The arc length is inversely proportional to the radius of the circular path, and it increases with the radius.

11. Consider a CD spinning clockwise. What is the sum of the instantaneous velocities of two points on both ends of its diameter?
   a. $2v$
   b. $\frac{v}{2}$
   c. $-v$
   d. 0

6.2 Uniform Circular Motion

12. What are the directions of the velocity and acceleration of an object in uniform circular motion?
   a. Velocity is tangential, and acceleration is radially outward.
   b. Velocity is tangential, and acceleration is radially inward.
   c. Velocity is radially outward, and acceleration is tangential.
   d. Velocity is radially inward, and acceleration is tangential.

13. Suppose you have an object tied to a rope and are rotating it over your head in uniform circular motion. If you increase the length of the rope, would you have to apply more or less force to maintain the same speed?
   a. More force is required, because the force is inversely proportional to the radius of the circular orbit.
   b. More force is required because the force is directly proportional to the radius of the circular orbit.
   c. Less force is required because the force is inversely proportional to the radius of the circular orbit.
   d. Less force is required because the force is directly proportional to the radius of the circular orbit.

6.3 Rotational Motion

14. Consider two spinning tops with different radii. Both have the same linear instantaneous velocities at their edges. Which top has a higher angular velocity?
   a. the top with the smaller radius because the radius of curvature is inversely proportional to the angular velocity
   b. the top with the smaller radius because the radius of curvature is directly proportional to the angular velocity
   c. the top with the larger radius because the radius of curvature is inversely proportional to the angular velocity
   d. The top with the larger radius because the radius of curvature is directly proportional to the angular velocity

15. A person tries to lift a stone by using a lever. If the lever arm is constant and the mass of the stone increases, what is true of the torque necessary to lift it?
   a. It increases, because the torque is directly proportional to the mass of the body.
   b. It increases because the torque is inversely proportional to the mass of the body.
   c. It decreases because the torque is directly proportional to the mass of the body.
   d. It decreases, because the torque is inversely proportional to the mass of the body.

Problems

6.1 Angle of Rotation and Angular Velocity

16. What is the angle of rotation (in degrees) between two hands of a clock, if the radius of the clock is $0.70\ \text{m}$ and the arc length separating the two hands is $1.0\ \text{m}$?
   a. 40°
   b. 80°
   c. 81°
   d. 163°

17. A clock has radius of $0.5\ \text{m}$. The outermost point on its minute hand travels along the edge. What is its tangential speed?
   a. $9 \times 10^{-4}\ \text{m/s}$
   b. $3.4 \times 10^{-3}\ \text{m/s}$
   c. $8.5 \times 10^{-4}\ \text{m/s}$
   d. $1.3 \times 10^{1}\ \text{m/s}$

6.2 Uniform Circular Motion

18. What is the centripetal force exerted on a 1,600 kg car that rounds a 100 m radius curve at 12 m/s?
   a. 192 N
   b. 1, 111 N
   c. 2, 300 N
   d. 13, 333 N

19. Find the frictional force between the tires and the road that allows a 1,000 kg car traveling at 30 m/s to round a 20 m radius curve.
   a. 22 N
   b. 667 N
   c. 1, 500 N
   d. 45, 000 N

6.3 Rotational Motion

20. An object's angular acceleration is 36 rad/s^2. If it were initially spinning with a velocity of 6.0 m/s, what would its angular velocity be after 5.0 s?
   a. 186 rad/s
   b. 190 rad/s^2
   c. –174 rad/s
   d. –174 rad/s^2

21. When a fan is switched on, it undergoes an angular acceleration of 150 rad/s^2. How long will it take to achieve its maximum angular velocity of 50 rad/s?
   a. –0.3 s
   b. 0.3 s
   c. 3.0 s

Performance Task

6.3 Rotational Motion

22. Design a lever arm capable of lifting a 0.5 kg object such as a stone. The force for lifting should be provided by placing coins on the other end of the lever. How many coins would you need? What happens if you shorten or lengthen the lever arm? What does this say about torque?

TEST PREP

Multiple Choice

6.1 Angle of Rotation and Angular Velocity

23. What is 1 radian approximately in degrees?
   a. 57.3°
   b. 360°
   c. π°
   d. $2\pi^\circ$

24. If the following objects are spinning at the same angular velocities, the edge of which one would have the highest speed?
   a. Mini CD
   b. Regular CD
   c. Vinyl record

25. What are possible units for tangential velocity?
   a. m/s
   b. rad/s
   c. °/s

26. What is 30° in radians?
   a. $\frac{\pi}{12}$
   b. $\frac{\pi}{9}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{3}$

27. For a given object, what happens to the arc length as the angle of rotation increases?
   a. The arc length is directly proportional to the angle of rotation, so it increases with the angle of rotation.
   b. The arc length is inversely proportional to the angle of rotation, so it decreases with the angle of rotation.
   c. The arc length is directly proportional to the angle of rotation, so it decreases with the angle of rotation.

d. The arc length is inversely proportional to the angle of rotation, so it increases with the angle of rotation.

6.2 Uniform Circular Motion

28. Which of these quantities is constant in uniform circular motion?
a. Speed
b. Velocity
c. Acceleration
d. Displacement

29. Which of these quantities impact centripetal force?
a. Mass and speed only
b. Mass and radius only
c. Speed and radius only
d. Mass, speed, and radius all impact centripetal force

30. An increase in the magnitude of which of these quantities causes a reduction in centripetal force?
a. Mass
b. Radius of curvature
c. Speed

31. What happens to centripetal acceleration as the radius of curvature decreases and the speed is constant, and why?
a. It increases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
b. It increases, because the centripetal acceleration is directly proportional to the radius of curvature.
c. It decreases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
d. It decreases, because the centripetal acceleration is directly proportional to the radius of the curvature.

32. Why do we experience more sideways acceleration while driving around sharper curves?
a. Centripetal acceleration is inversely proportional to the radius of curvature, so it increases as the radius of curvature decreases.
b. Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature decreases.
c. Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature increases.
d. Centripetal acceleration is directly proportional to the radius of curvature, so it increases as the radius of curvature increases.

6.3 Rotational Motion

33. Which of these quantities is not described by the kinematics of rotational motion?
a. Rotation angle
b. Angular acceleration
c. Centripetal force
d. Angular velocity

34. In the equation $\tau = rF\sin\theta$, what is F?
a. Linear force
b. Centripetal force
c. Angular force

35. What happens when two torques act equally in opposite directions?
a. Angular velocity is zero.
b. Angular acceleration is zero.

36. What is the mathematical relationship between angular and linear accelerations?
a. $a = r\alpha$
b. $a = \frac{\alpha}{r}$
c. $a = r^2\alpha$
d. $a = \frac{\alpha}{r^2}$

Short Answer

6.1 Angle of Rotation and Angular Velocity

37. What is the rotational analog of linear velocity?
a. Angular displacement
b. Angular velocity
c. Angular acceleration
d. Angular momentum

38. What is the rotational analog of distance?
a. Rotational angle
b. Torque
c. Angular velocity
d. Angular momentum

39. What is the equation that relates the linear speed of a point on a rotating object with the object's angular quantities?
a. $v = \frac{\omega}{r}$
b. $v = r\omega$
c. $v = \frac{\alpha}{r}$
d. $v = r\alpha$

40. As the angular velocity of an object increases, what happens to the linear velocity of a point on that object?
a. It increases, because linear velocity is directly proportional to angular velocity.
b. It increases, because linear velocity is inversely proportional to angular velocity.

c. It decreases because linear velocity is directly proportional to angular velocity.
d. It decreases because linear velocity is inversely proportional to angular velocity.

41. What is angular speed in terms of tangential speed and the radius?
a. $\omega = \frac{v^2}{r}$
b. $\omega = \frac{v}{r}$
c. $\omega = rv$
d. $\omega = rv^2$

42. Why are radians dimensionless?
a. Radians are dimensionless, because they are defined as a ratio of distances. They are defined as the ratio of the arc length to the radius of the circle.
b. Radians are dimensionless because they are defined as a ratio of distances. They are defined as the ratio of the area to the radius of the circle.
c. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the arc length to the radius of the circle.
d. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the area to the radius of the circle.

6.2 Uniform Circular Motion

43. What type of quantity is centripetal acceleration?
a. Scalar quantity; centripetal acceleration has magnitude only but no direction
b. Scalar quantity; centripetal acceleration has magnitude as well as direction
c. Vector quantity; centripetal acceleration has magnitude only but no direction
d. Vector quantity; centripetal acceleration has magnitude as well as direction

44. What are the standard units for centripetal acceleration?
a. m/s
b. m/s^2
c. m^2/s
d. m^2/s^2

45. What is the angle formed between the vectors of tangential velocity and centripetal force?
a. 0°
b. 30°
c. 90°
d. 180°

46. What is the angle formed between the vectors of centripetal acceleration and centripetal force?
a. 0°
b. 30°
c. 90°
d. 180°

47. What are the standard units for centripetal force?
a. m
b. m/s
c. m/s^2
d. newtons

48. As the mass of an object in uniform circular motion increases, what happens to the centripetal force required to keep it moving at the same speed?
a. It increases, because the centripetal force is directly proportional to the mass of the rotating body.
b. It increases, because the centripetal force is inversely proportional to the mass of the rotating body.
c. It decreases, because the centripetal force is directly proportional to the mass of the rotating body.
d. It decreases, because the centripetal force is inversely proportional to the mass of the rotating body.

6.3 Rotational Motion

49. The relationships between which variables are described by the kinematics of rotational motion?
a. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, and angular acceleration.
b. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and angular momentum.
c. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
d. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, torque, and time.

50. What is the kinematics relationship between ω, α, and t?
a. $\omega = \alpha t$
b. $\omega = \omega_0 - \alpha t$
c. $\omega = \omega_0 + \alpha t$
d. $\omega = \omega_0 + \frac{1}{2}\alpha t$

51. What kind of quantity is torque?
a. Scalar
b. Vector

c. Dimensionless
d. Fundamental quantity

52. If a linear force is applied to a lever arm farther away from the pivot point, what happens to the resultant torque?
a. It decreases.
b. It increases.
c. It remains the same.
d. It changes the direction.

53. How can the same force applied to a lever produce different torques?
a. By applying the force at different points of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
b. By applying the force at the same point of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
c. By applying the force at different points of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.
d. By applying the force at the same point of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.

Extended Response

6.1 Angle of Rotation and Angular Velocity

54. Consider two pits on a CD, one close to the center and one close to the outer edge. When the CD makes one full rotation, which pit would have gone through a greater angle of rotation? Which one would have covered a greater arc length?
a. The one close to the center would go through the greater angle of rotation. The one near the outer edge would trace a greater arc length.
b. The one close to the center would go through the greater angle of rotation. The one near the center would trace a greater arc length.
c. Both would go through the same angle of rotation. The one near the outer edge would trace a greater arc length.
d. Both would go through the same angle of rotation. The one near the center would trace a greater arc length.

55. Consider two pits on a CD, one close to the center and one close to the outer edge. For a given angular velocity of the CD, which pit has a higher angular velocity? Which has a higher tangential velocity?
a. The point near the center would have the greater angular velocity and the point near the outer edge would have the higher linear velocity.
b. The point near the edge would have the greater angular velocity and the point near the center would have the higher linear velocity.
c. Both have the same angular velocity and the point near the outer edge would have the higher linear velocity.
d. Both have the same angular velocity and the point near the center would have the higher linear velocity.

56. What happens to tangential velocity as the radius of an object increases provided the angular velocity remains the same?
a. It increases because tangential velocity is directly proportional to the radius.
b. It increases because tangential velocity is inversely proportional to the radius.
c. It decreases because tangential velocity is directly proportional to the radius.
d. It decreases because tangential velocity is inversely proportional to the radius.

6.2 Uniform Circular Motion

57. Is an object in uniform circular motion accelerating? Why or why not?
a. Yes, because the velocity is not constant.
b. No, because the velocity is not constant.
c. Yes, because the velocity is constant.
d. No, because the velocity is constant.

58. An object is in uniform circular motion. Suppose the centripetal force was removed. In which direction would the object now travel?
a. In the direction of the centripetal force
b. In the direction opposite to the direction of the centripetal force
c. In the direction of the tangential velocity
d. In the direction opposite to the direction of the tangential velocity

59. An object undergoes uniform circular motion. If the radius of curvature and mass of the object are constant, what is the centripetal force proportional to?
a. $F_c \propto \frac{1}{v}$
b. $F_c \propto \frac{1}{v^2}$
c. $F_c \propto v$
d. $F_c \propto v^2$

6.3 Rotational Motion

60. Why do tornadoes produce more wind speed at the

bottom of the funnel?

a. Wind speed is greater at the bottom because rate of rotation increases as the radius increases.
b. Wind speed is greater at the bottom because rate of rotation increases as the radius decreases.
c. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.
d. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.

61. How can you maximize the torque applied to a given lever arm without applying more force?

a. The force should be applied perpendicularly to the lever arm as close as possible from the pivot point.
b. The force should be applied perpendicularly to the lever arm as far as possible from the pivot point.
c. The force should be applied parallel to the lever arm as far as possible from the pivot point.
d. The force should be applied parallel to the lever arm as close as possible from the pivot point.

62. When will an object continue spinning at the same angular velocity?

a. When net torque acting on it is zero
b. When net torque acting on it is non zero
c. When angular acceleration is positive
d. When angular acceleration is negative