GRASP CHECK

Based on the equation $\mathbf{L} = I\omega$, how would you expect the moment of inertia of an object to affect angular momentum? How would angular velocity affect angular momentum?

a. Large moment of inertia implies large angular momentum, and large angular velocity implies large angular momentum.
b. Large moment of inertia implies small angular momentum, and large angular velocity implies small angular momentum.
c. Large moment of inertia implies large angular momentum, and large angular velocity implies small angular momentum.
d. Large moment of inertia implies small angular momentum, and large angular velocity implies large angular momentum.

Check Your Understanding

7. When is momentum said to be conserved?
   a. When momentum is changing during an event
   b. When momentum is increasing during an event
   c. When momentum is decreasing during an event
   d. When momentum is constant throughout an event

8. A ball is hit by a racket and its momentum changes. How is momentum conserved in this case?
   a. Momentum of the system can never be conserved in this case.
   b. Momentum of the system is conserved if the momentum of the racket is not considered.
   c. Momentum of the system is conserved if the momentum of the racket is also considered.
   d. Momentum of the system is conserved if the momenta of the racket and the player are also considered.

9. State the law of conservation of momentum.
   a. Momentum is conserved for an isolated system with any number of objects in it.
   b. Momentum is conserved for an isolated system with an even number of objects in it.
   c. Momentum is conserved for an interacting system with any number of objects in it.
   d. Momentum is conserved for an interacting system with an even number of objects in it.

8.3 Elastic and Inelastic Collisions

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Distinguish between elastic and inelastic collisions
- Solve collision problems by applying the law of conservation of momentum

Section Key Terms

elastic collision	inelastic collision	point masses	recoil

Elastic and Inelastic Collisions

When objects collide, they can either stick together or bounce off one another, remaining separate. In this section, we'll cover these two different types of collisions, first in one dimension and then in two dimensions.

In an **elastic collision**, the objects separate after impact and don't lose any of their kinetic energy. Kinetic energy is the energy of motion and is covered in detail elsewhere. The law of conservation of momentum is very useful here, and it can be used whenever the net external force on a system is zero. Figure 8.6 shows an elastic collision where momentum is conserved.

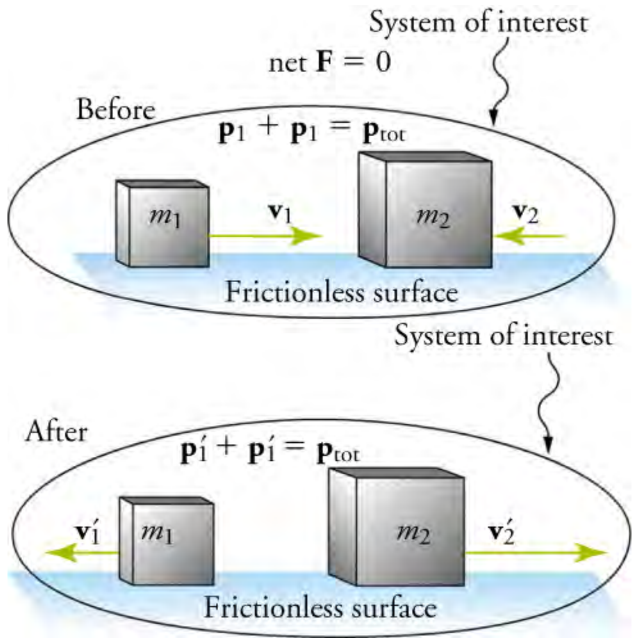

Figure 8.6 The diagram shows a one-dimensional elastic collision between two objects.

An animation of an elastic collision between balls can be seen by watching this [video (http://openstax.org/l/28elasticball)](http://openstax.org/l/28elasticball) . It replicates the elastic collisions between balls of varying masses.

Perfectly elastic collisions can happen only with subatomic particles. Everyday observable examples of perfectly elastic collisions don't exist—some kinetic energy is always lost, as it is converted into heat transfer due to friction. However, collisions between everyday objects are almost perfectly elastic when they occur with objects and surfaces that are nearly frictionless, such as with two steel blocks on ice.

Now, to solve problems involving one-dimensional elastic collisions between two objects, we can use the equation for conservation of momentum. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2 (\mathbf{F}_{\text{net}} = 0).$$

Substituting the definition of momentum $\mathbf{p} = m\mathbf{v}$ for each initial and final momentum, we get

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2,$$

where the primes (') indicate values after the collision; In some texts, you may see *i* for initial (before collision) and *f* for final (after collision). The equation assumes that the mass of each object does not change during the collision.

WATCH PHYSICS

Momentum: Ice Skater Throws a Ball

This video covers an elastic collision problem in which we find the ***recoil** velocity* of an ice skater who throws a ball straight forward. To clarify, Sal is using the equation

$$m_{\text{ball}} \mathbf{V}_{\text{ball}} + m_{\text{skater}} \mathbf{V}_{\text{skater}} = m_{\text{ball}} \mathbf{v}'_{\text{ball}} + m_{\text{skater}} \mathbf{v}'_{\text{skater}}.$$

[Click to view content (https://www.khanacademy.org/embed_video?v=vPkkCOlGND4)](https://www.khanacademy.org/embed_video?v=vPkkCOlGND4)

GRASP CHECK

The resultant vector of the addition of vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}$. The magnitudes of $\vec{a}$, $\vec{b}$, and $\vec{r}$ are A, B, and R, respectively. Which of the following is true?

a. $R_x + R_y = 0$
b. $A_x + A_y = \vec{A}$

c. $A_x + B_y = B_x + A_y$
d. $A_x + B_x = R_x$

Now, let us turn to the second type of collision. An **inelastic collision** is one in which objects stick together after impact, and kinetic energy is *not* conserved. This lack of conservation means that the forces between colliding objects may convert kinetic energy to other forms of energy, such as potential energy or thermal energy. The concepts of energy are discussed more thoroughly elsewhere. For inelastic collisions, kinetic energy may be lost in the form of heat. Figure 8.7 shows an example of an inelastic collision. Two objects that have equal masses head toward each other at equal speeds and then stick together. The two objects come to rest after sticking together, conserving momentum but not kinetic energy after they collide. Some of the energy of motion gets converted to thermal energy, or heat.

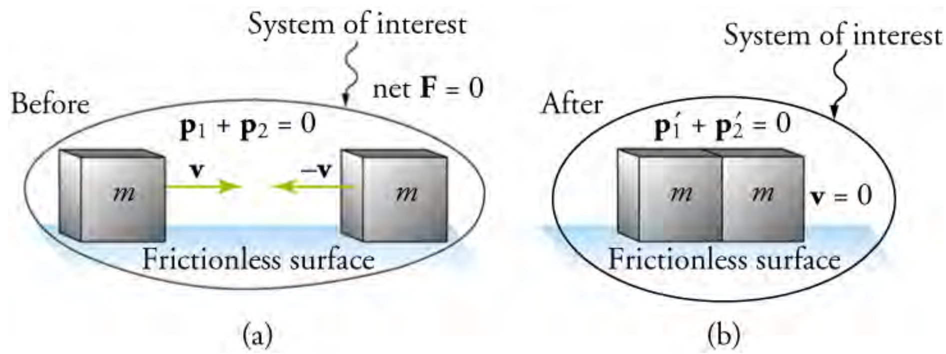

Figure 8.7 A one-dimensional inelastic collision between two objects. Momentum is conserved, but kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward each other at the same speed. (b) The objects stick together, creating a perfectly inelastic collision. In the case shown in this figure, the combined objects stop; This is not true for all inelastic collisions.

Since the two objects stick together after colliding, they move together at the same speed. This lets us simplify the conservation of momentum equation from

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v'}_1 + m_2 \mathbf{v'}_2$$

to

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v'}$$

for inelastic collisions, where **v**' is the final velocity for both objects as they are stuck together, either in motion or at rest.

WATCH PHYSICS

Introduction to Momentum

This video reviews the definitions of momentum and impulse. It also covers an example of using conservation of momentum to solve a problem involving an inelastic collision between a car with constant velocity and a stationary truck. Note that Sal accidentally gives the unit for impulse as Joules; it is actually N · s or k · gm/s.

Click to view content (https://www.khanacademy.org/embed_video?v=XFhntPxowoU)

GRASP CHECK

How would the final velocity of the car-plus-truck system change if the truck had some initial velocity moving in the same direction as the car? What if the truck were moving in the opposite direction of the car initially? Why?

a. If the truck was initially moving in the same direction as the car, the final velocity would be greater. If the truck was initially moving in the opposite direction of the car, the final velocity would be smaller.
b. If the truck was initially moving in the same direction as the car, the final velocity would be smaller. If the truck was initially moving in the opposite direction of the car, the final velocity would be greater.
c. The direction in which the truck was initially moving would not matter. If the truck was initially moving in either

direction, the final velocity would be smaller.

d. The direction in which the truck was initially moving would not matter. If the truck was initially moving in either direction, the final velocity would be greater.

Snap Lab

Ice Cubes and Elastic Collisions

In this activity, you will observe an elastic collision by sliding an ice cube into another ice cube on a smooth surface, so that a negligible amount of energy is converted to heat.

- Several ice cubes (The ice must be in the form of cubes.)
- A smooth surface

Procedure

1. Find a few ice cubes that are about the same size and a smooth kitchen tabletop or a table with a glass top.
2. Place the ice cubes on the surface several centimeters away from each other.
3. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes.
4. Explain the speeds and directions of the ice cubes using momentum.

GRASP CHECK

Was the collision elastic or inelastic?

a. perfectly elastic
b. perfectly inelastic
c. Nearly perfect elastic
d. Nearly perfect inelastic

TIPS FOR SUCCESS

Here's a trick for remembering which collisions are elastic and which are inelastic: Elastic is a bouncy material, so when objects *bounce* off one another in the collision and separate, it is an elastic collision. When they don't, the collision is inelastic.

Solving Collision Problems

The Khan Academy videos referenced in this section show examples of elastic and inelastic collisions in one dimension. In one-dimensional collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and just as we did with two-dimensional forces, we will solve these problems by first choosing a coordinate system and separating the motion into its x and y components.

One complication with two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass each other, they will spin in circles. We will not consider such rotation until later, and so for now, we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{net} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in Figure 8.8. Because momentum is conserved, the components of momentum along the x- and y-axes, displayed as $\mathbf{p}_x$ and $\mathbf{p}_y$, will also be conserved. With the chosen coordinate system, $\mathbf{p}_y$ is initially zero and $\mathbf{p}_x$ is the momentum of the incoming particle.

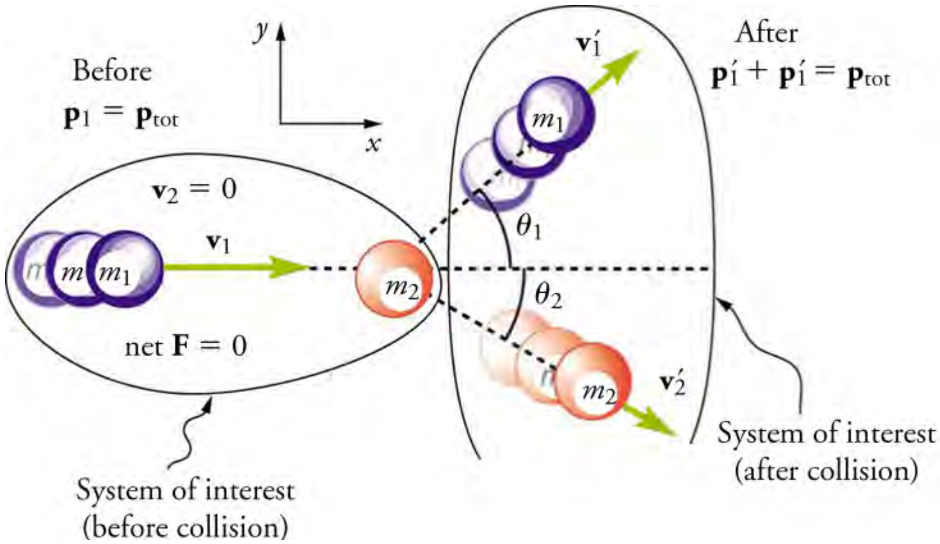

Figure 8.8 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and $\mathbf{v}_1$ is parallel to the x-axis.

Now, we will take the conservation of momentum equation, $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ and break it into its x and y components.

Along the x-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1x} + \mathbf{p}_{2x} = \mathbf{p}'_{1x} + \mathbf{p}'_{2x}.$$

In terms of masses and velocities, this equation is

$$m_1 \mathbf{v}_{1\ x} + m_2 \mathbf{v}_{2\ x} = m_1 \mathbf{v}'_{1\ x} + m_2 \mathbf{v}'_{2\ x}. \tag{8.3}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 \mathbf{v}_{1\ x} = m_1 \mathbf{v}'_{1\ x} + m_2 \mathbf{v}'_{2\ x}. \tag{8.4}$$

The components of the velocities along the x-axis have the form $\mathbf{v} \cos \theta$. Because particle 1 initially moves along the x-axis, we find $\mathbf{v}_{1x} = \mathbf{v}_1$. Conservation of momentum along the x-axis gives the equation

$$m_1 \mathbf{v}_1 = m_1 \mathbf{v}'_1 \cos \theta_1 + m_2 \mathbf{v}'_2 \cos \theta_2,$$

where θ_1 and θ_2 are as shown in Figure 8.8.

Along the y-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1\ y} + \mathbf{p}_{2\ y} = \mathbf{p}'_{1\ y} + \mathbf{p}'_{2\ y}, \tag{8.5}$$

or

$$m_1 \mathbf{v}_{1\ y} + m_2 \mathbf{v}_{2\ y} = m_1 \mathbf{v}'_{1\ y} + m_2 \mathbf{v}'_{2\ y}. \tag{8.6}$$

But $\mathbf{v}_1 y$ is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, $\mathbf{v}_2 y$ is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 \mathbf{v}'_1 y + m_2 \mathbf{v}'_2 y. \tag{8.7}$$

The components of the velocities along the y-axis have the form $\mathbf{v} \sin \theta$. Therefore, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 \mathbf{v}'_1 \sin \theta_1 + m_2 \mathbf{v}'_2 \sin \theta_2$$

Virtual Physics

Collision Lab

In this simulation, you will investigate collisions on an air hockey table. Place checkmarks next to the momentum vectors and momenta diagram options. Experiment with changing the masses of the balls and the initial speed of ball 1. How does this affect the momentum of each ball? What about the total momentum? Next, experiment with changing the elasticity of the collision. You will notice that collisions have varying degrees of elasticity, ranging from perfectly elastic to perfectly inelastic.

Click to view content (https://archive.cnx.org/specials/2c7acb3c-2fbd-11e5-b2d9-e7f92291703c/collision-lab/)

GRASP CHECK

If you wanted to maximize the velocity of ball 2 after impact, how would you change the settings for the masses of the balls, the initial speed of ball 1, and the elasticity setting? Why? Hint—Placing a checkmark next to the velocity vectors and removing the momentum vectors will help you visualize the velocity of ball 2, and pressing the More Data button will let you take readings.

a. Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 50 percent.
b. Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 100 percent.
c. Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 100 percent.
d. Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 50 percent.

WORKED EXAMPLE

Calculating Velocity: Inelastic Collision of a Puck and a Goalie

Find the recoil velocity of a 70 kg ice hockey goalie who catches a 0.150-kg hockey puck slapped at him at a velocity of 35 m/s. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible (see Figure 8.9).

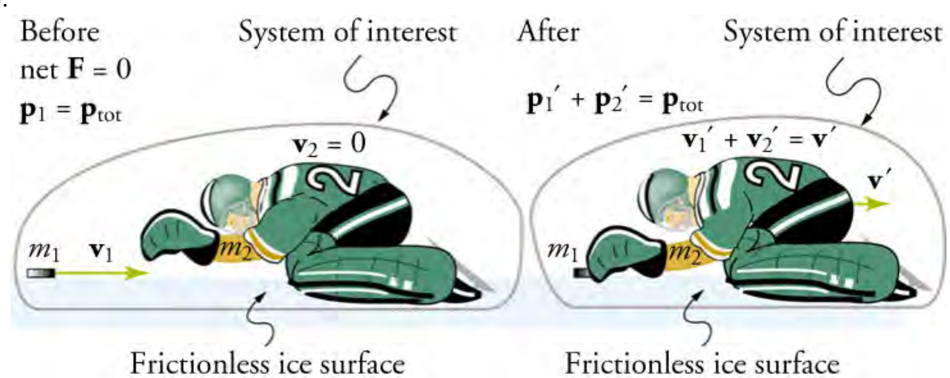

Figure 8.9 An ice hockey goalie catches a hockey puck and recoils backward in an inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. Therefore, we can use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same.

Solution

For an inelastic collision, conservation of momentum is

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v}', \tag{8.8}$$

where $\mathbf{v}'$ is the velocity of both the goalie and the puck after impact. Because the goalie is initially at rest, we know $\mathbf{v}_2 = 0$. This simplifies the equation to

$$m_1 \mathbf{v}_1 = (m_1 + m_2) \mathbf{v}'. \tag{8.9}$$

Solving for $\mathbf{v}'$ yields

$$\mathbf{v}' = \left(\frac{m_1}{m_1 + m_2}\right)\mathbf{v}_1. \quad (8.10)$$

Entering known values in this equation, we get

$$\begin{aligned}\mathbf{v}' &= \left(\frac{0.150\ \text{kg}}{70.0\ \text{kg}+0.150\ \text{kg}}\right)(35\ \text{m/s}) \\ &= 7.48 \times 10^{-2}\,\text{m/s}.\end{aligned} \quad (8.11)$$

Discussion

This recoil velocity is small and in the same direction as the puck's original velocity.

WORKED EXAMPLE

Calculating Final Velocity: Elastic Collision of Two Carts

Two hard, steel carts collide head-on and then ricochet off each other in opposite directions on a frictionless surface (see Figure 8.10). Cart 1 has a mass of 0.350 kg and an initial velocity of 2 m/s. Cart 2 has a mass of 0.500 kg and an initial velocity of −0.500 m/s. After the collision, cart 1 recoils with a velocity of −4 m/s. What is the final velocity of cart 2?

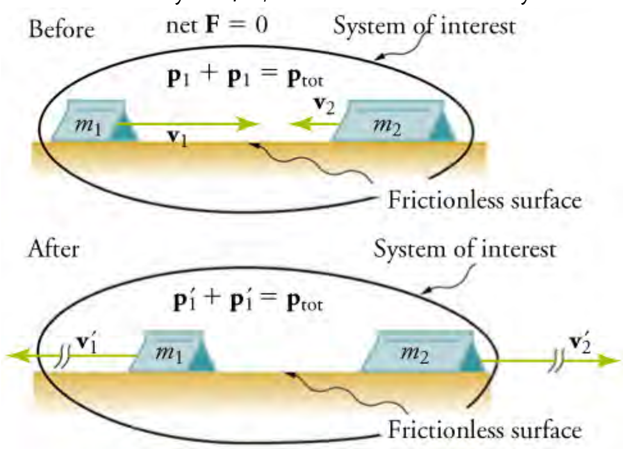

Figure 8.10 Two carts collide with each other in an elastic collision.

Strategy

Since the track is frictionless, $\mathbf{F}_{\text{net}} = 0$ and we can use conservation of momentum to find the final velocity of cart 2.

Solution

As before, the equation for conservation of momentum for a one-dimensional elastic collision in a two-object system is

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2. \quad (8.12)$$

The only unknown in this equation is $\mathbf{v}'_2$. Solving for $\mathbf{v}'_2$ and substituting known values into the previous equation yields

$$\begin{aligned}\mathbf{v}'_2 &= \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 - m_1\mathbf{v}'_1}{m_2} \\ &= \frac{(0.350\text{kg})(2.00\text{m/s}) + (0.500\ \text{kg})(-0.500\ \text{m/s}) - (0.350\text{kg})(-4.00\text{m/s})}{0.500\text{kg}} \\ &= 3.70\text{m/s}.\end{aligned} \quad (8.13)$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision.

WORKED EXAMPLE

Calculating Final Velocity in a Two-Dimensional Collision

Suppose the following experiment is performed (Figure 8.11). An object of mass 0.250 kg (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object of mass 0.400 kg (m_2). The 0.250 kg object emerges from the room at an angle of 45° with its incoming direction. The speed of the 0.250 kg object is originally 2 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2 and θ_2) of the 0.400 kg object after the collision.

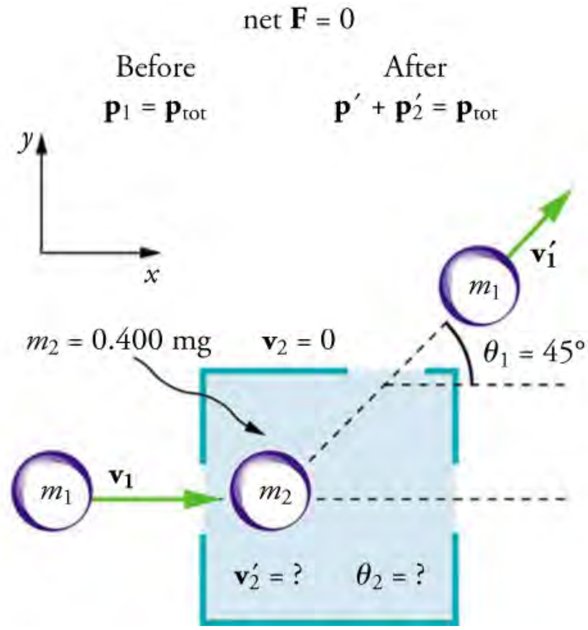

Figure 8.11 The incoming object of mass m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which the object of mass m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Strategy

Momentum is conserved because the surface is frictionless. We chose the coordinate system so that the initial velocity is parallel to the *x*-axis, and conservation of momentum along the *x*- and *y*-axes applies.

Everything is known in these equations except $\mathbf{v}'_2$ and θ_2, which we need to find. We can find two unknowns because we have two independent equations—the equations describing the conservation of momentum in the *x* and *y* directions.

Solution

First, we'll solve both conservation of momentum equations ($m_1\mathbf{v}_1 = m_1\mathbf{v'}_1 \cos\theta_1 + m_2\mathbf{v'}_2 \cos\theta_2$ and $0 = m_1\mathbf{v'}_1 \sin\theta_1 + m_2\mathbf{v'}_2 \sin\theta_2$) for $\mathbf{v}'_2 \sin\theta_2$.

For conservation of momentum along x-axis, let's substitute $\sin\theta_2 / \tan\theta_2$ for $\cos\theta_2$ so that terms may cancel out later on. This comes from rearranging the definition of the trigonometric identity $\tan\theta = \sin\theta / \cos\theta$. This gives us

$$m_1\mathbf{v}_1 = m_1\mathbf{v'}_1 \cos\theta_1 + m_2\mathbf{v'}_2 \frac{\sin\theta_2}{\tan\theta_2}. \tag{8.14}$$

Solving for $\mathbf{v}'_2 \sin\theta_2$ yields

$$\mathbf{v'}_2 \sin\theta_2 = \frac{(m_1\mathbf{v}_1 - m_1\mathbf{v'}_1 \cos\theta_1)(\tan\theta_2)}{m_2}. \tag{8.15}$$

For conservation of momentum along *y*-axis, solving for $\mathbf{v}'_2 \sin\theta_2$ yields

$$\mathbf{v'}_2 \sin\theta_2 = \frac{-(m_1\mathbf{v'}_1 \sin\theta_1)}{m_2}. \tag{8.16}$$

Since both equations equal $\mathbf{v}'_2 \sin\theta_2$, we can set them equal to one another, yielding

$$\frac{(m_1\mathbf{v}_1 - m_1\mathbf{v}'_1\cos\theta_1)(\tan\theta_2)}{m_2} = \frac{-(m_1\mathbf{v}'_1\sin\theta_1)}{m_2}. \quad (8.17)$$

Solving this equation for $\tan\theta_2$, we get

$$\tan\theta_2 = \frac{\mathbf{v}'_1\sin\theta_1}{\mathbf{v}'_1\cos\theta_1 - \mathbf{v}_1}. \quad (8.18)$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50)(0.707)}{(1.50)(0.707) - 2.00} = -1.129\ . \quad (8.19)$$

Therefore,

$$\theta_2 = \tan^{-1}(-1.129) = 312^0. \quad (8.20)$$

Since angles are defined as positive in the counterclockwise direction, m_2 is scattered to the right.

We'll use the conservation of momentum along the y-axis equation to solve for $\mathbf{v}'_2$.

$$\mathbf{v}'_2 = -\frac{m_1}{m_2}\mathbf{v}'_1\frac{\sin\theta_1}{\sin\theta_2} \quad (8.21)$$

Entering known values into this equation gives

$$\mathbf{v}'_2 = -\frac{(0.250)}{(0.400)}(1.50)\left(\frac{0.7071}{-0.7485}\right). \quad (8.22)$$

Therefore,

$$\mathbf{v}'_2 = 0.886\ \text{m/s}. \quad (8.23)$$

Discussion

Either equation for the *x*- or *y*-axis could have been used to solve for $\mathbf{v}'_2$, but the equation for the *y*-axis is easier because it has fewer terms.

Practice Problems

10. In an elastic collision, an object with momentum $25\ \text{kg}\cdot\text{m/s}$ collides with another object moving to the right that has a momentum $35\ \text{kg}\cdot\text{m/s}$. After the collision, both objects are still moving to the right, but the first object's momentum changes to $10\ \text{kg}\cdot\text{m/s}$. What is the final momentum of the second object?

a. $10\ \text{kg}\cdot\text{m/s}$
b. $20\ \text{kg}\cdot\text{m/s}$
c. $35\ \text{kg}\cdot\text{m/s}$
d. $50\ \text{kg}\cdot\text{m/s}$

11. In an elastic collision, an object with momentum 25 kg · m/s collides with another that has a momentum 35 kg · m/s. The first object's momentum changes to 10 kg · m/s. What is the final momentum of the second object?

a. 10 kg · m/s
b. 20 kg · m/s
c. 35 kg · m/s
d. 50 kg · m/s

Check Your Understanding

12. What is an elastic collision?

a. An elastic collision is one in which the objects after impact are deformed permanently.
b. An elastic collision is one in which the objects after impact lose some of their internal kinetic energy.

c. An elastic collision is one in which the objects after impact do not lose any of their internal kinetic energy.
d. An elastic collision is one in which the objects after impact become stuck together and move with a common velocity.

13. Are perfectly elastic collisions possible?
a. Perfectly elastic collisions are not possible.
b. Perfectly elastic collisions are possible only with subatomic particles.
c. Perfectly elastic collisions are possible only when the objects stick together after impact.
d. Perfectly elastic collisions are possible if the objects and surfaces are nearly frictionless.

14. What is the equation for conservation of momentum for two objects in a one-dimensional collision?
a. $\mathbf{p}_1 + \mathbf{p}_1{}' = \mathbf{p}_2 + \mathbf{p}_2{}'$
b. $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1{}' + \mathbf{p}_2{}'$
c. $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{p}_1{}' - \mathbf{p}_2{}'$
d. $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_1{}' + \mathbf{p}_2{}' = 0$