constant?

a. $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{v}}{\Delta m \Delta t}$
b. $\mathbf{F}_{\text{net}} = \frac{m \Delta t}{\Delta \mathbf{v}}$
c. $\mathbf{F}_{\text{net}} = \frac{m \Delta \mathbf{v}}{\Delta t}$
d. $\mathbf{F}_{\text{net}} = \frac{\Delta m \Delta \mathbf{v}}{\Delta t}$

6. Give an example of a system whose mass is not constant.

a. A spinning top
b. A baseball flying through the air
c. A rocket launched from Earth
d. A block sliding on a frictionless inclined plane

8.2 Conservation of Momentum

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the law of conservation of momentum verbally and mathematically

Section Key Terms

angular momentum	isolated system	law of conservation of momentum

Conservation of Momentum

It is important we realize that momentum is conserved during collisions, explosions, and other events involving objects in motion. To say that a quantity is conserved means that it is constant throughout the event. In the case of conservation of momentum, the total momentum in the system remains the same before and after the collision.

You may have noticed that momentum was *not* conserved in some of the examples previously presented in this chapter. where forces acting on the objects produced large changes in momentum. Why is this? The systems of interest considered in those problems were not inclusive enough. If the systems were expanded to include more objects, then momentum would in fact be conserved in those sample problems. It is always possible to find a larger system where momentum is conserved, even though momentum changes for individual objects within the system.

For example, if a football player runs into the goalpost in the end zone, a force will cause him to bounce backward. His momentum is obviously greatly changed, and considering only the football player, we would find that momentum is not conserved. However, the system can be expanded to contain the entire Earth. Surprisingly, Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. The effect on Earth is not noticeable because it is so much more massive than the player, but the effect is real.

Next, consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—in the example shown in Figure 8.4 of one car bumping into another. Both cars are coasting in the same direction when the lead car, labeled m_2, is bumped by the trailing car, labeled m_1. The only unbalanced force on each car is the force of the collision, assuming that the effects due to friction are negligible. Car *m1* slows down as a result of the collision, losing some momentum, while car *m2* speeds up and gains some momentum. If we choose the system to include both cars and assume that friction is negligible, then the momentum of the two-car system should remain constant. Now we will prove that the total momentum of the two-car system does in fact remain constant, and is therefore conserved.

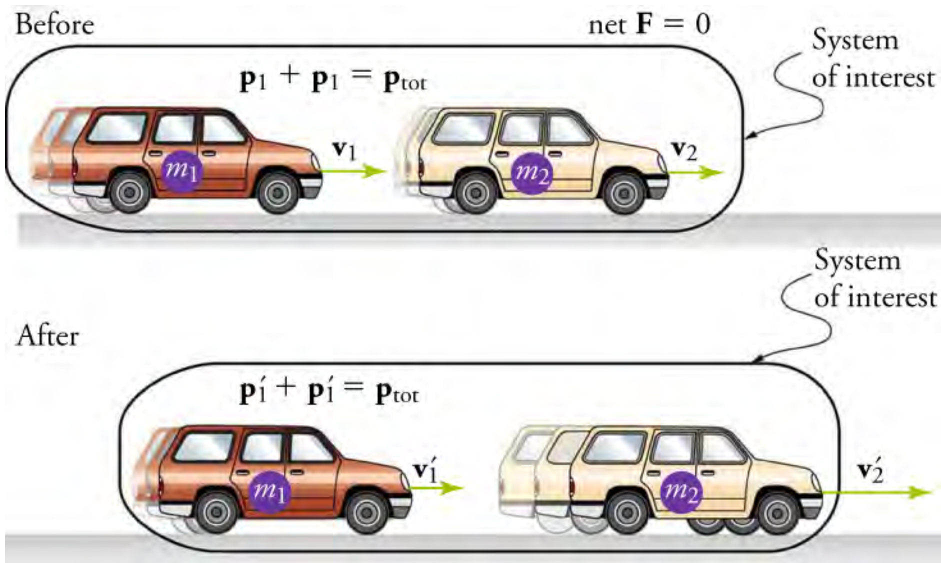

Figure 8.4 Car of mass m_1 moving with a velocity of $\mathbf{v}_1$ bumps into another car of mass m_2 and velocity $\mathbf{v}_2$. As a result, the first car slows down to a velocity of $\mathbf{v}'_1$ and the second speeds up to a velocity of $\mathbf{v}'_2$. The momentum of each car is changed, but the total momentum $\mathbf{p}_{tot}$ of the two cars is the same before and after the collision if you assume friction is negligible.

Using the impulse-momentum theorem, the change in momentum of car 1 is given by

$$\Delta \mathbf{p}_1 = \mathbf{F}_1 \Delta t,$$

where $\mathbf{F}_1$ is the force on car 1 due to car 2, and Δt is the time the force acts, or the duration of the collision.

Similarly, the change in momentum of car 2 is $\Delta \mathbf{p}_2 = \mathbf{F}_2 \Delta t$ where $\mathbf{F}_2$ is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law of motion that $\mathbf{F}_2 = -\mathbf{F}_1$, and so $\Delta \mathbf{p}_2 = -\mathbf{F}_1 \Delta t = -\Delta \mathbf{p}_1$.

Therefore, the changes in momentum are equal and opposite, and $\Delta \mathbf{p}_1 + \Delta \mathbf{p}_2 = 0$.

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p'}_1 + \mathbf{p'}_2,$$

where $\mathbf{p}'_1$ and $\mathbf{p}'_2$ are the momenta of cars 1 and 2 after the collision.

This result that momentum is conserved is true not only for this example involving the two cars, but for any system where the net external force is zero, which is known as an **isolated system**. The **law of conservation of momentum** states that for an isolated system with any number of objects in it, the total momentum is conserved. In equation form, the law of conservation of momentum for an isolated system is written as

$$\mathbf{p}_{tot} = \text{constant}$$

or

$$\mathbf{p}_{tot} = \mathbf{p'}_{tot},$$

where $\mathbf{p}_{tot}$ is the total momentum, or the sum of the momenta of the individual objects in the system at a given time, and $\mathbf{p}'_{tot}$ is the total momentum some time later.

The conservation of momentum principle can be applied to systems as diverse as a comet striking the Earth or a gas containing huge numbers of atoms and molecules. Conservation of momentum appears to be violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

TIPS FOR SUCCESS

Momenta is the plural form of the word momentum. One object is said to have momentum, but two or more objects are said to have *momenta*.

FUN IN PHYSICS

Angular Momentum in Figure Skating

So far we have covered linear momentum, which describes the inertia of objects traveling in a straight line. But we know that many objects in nature have a curved or circular path. Just as linear motion has linear momentum to describe its tendency to move forward, circular motion has the equivalent **angular momentum** to describe how rotational motion is carried forward.

This is similar to how torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass or inertia. You may recall learning that the quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m at a distance r from the center of rotation.

We already know the equation for linear momentum, $\mathbf{p} = m\mathbf{v}$. Since angular momentum is analogous to linear momentum, the moment of inertia (I) is analogous to mass, and angular velocity ($\boldsymbol{\omega}$) is analogous to linear velocity, it makes sense that angular momentum ($\mathbf{L}$) is defined as

$$\mathbf{L} = I\boldsymbol{\omega}$$

Angular momentum is conserved when the net external torque ($\boldsymbol{\tau}$) is zero, just as linear momentum is conserved when the net external force is zero.

Figure skaters take advantage of the conservation of angular momentum, likely without even realizing it. In Figure 8.5, a figure skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice, and because the friction is exerted very close to the pivot point. Both $\mathbf{F}$ and r are small, and so $\boldsymbol{\tau}$ is negligibly small.

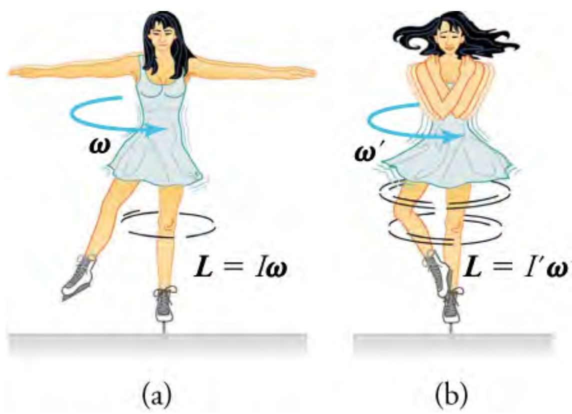

Figure 8.5 (a) An ice skater is spinning on the tip of her skate with her arms extended. In the next image, (b), her rate of spin increases greatly when she pulls in her arms.

Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that $\mathbf{L} = \mathbf{L}'$.

Expressing this equation in terms of the moment of inertia,

$$I\boldsymbol{\omega} = I'\boldsymbol{\omega}',$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity $\boldsymbol{\omega}'$ must increase to keep the angular momentum constant. This allows her to spin much faster without exerting any extra torque.

A video (http://openstax.org/l/28figureskater) is also available that shows a real figure skater executing a spin. It discusses the physics of spins in figure skating.

GRASP CHECK

Based on the equation $\mathbf{L} = I\omega$, how would you expect the moment of inertia of an object to affect angular momentum? How would angular velocity affect angular momentum?

a. Large moment of inertia implies large angular momentum, and large angular velocity implies large angular momentum.
b. Large moment of inertia implies small angular momentum, and large angular velocity implies small angular momentum.
c. Large moment of inertia implies large angular momentum, and large angular velocity implies small angular momentum.
d. Large moment of inertia implies small angular momentum, and large angular velocity implies large angular momentum.

Check Your Understanding

7. When is momentum said to be conserved?
   a. When momentum is changing during an event
   b. When momentum is increasing during an event
   c. When momentum is decreasing during an event
   d. When momentum is constant throughout an event

8. A ball is hit by a racket and its momentum changes. How is momentum conserved in this case?
   a. Momentum of the system can never be conserved in this case.
   b. Momentum of the system is conserved if the momentum of the racket is not considered.
   c. Momentum of the system is conserved if the momentum of the racket is also considered.
   d. Momentum of the system is conserved if the momenta of the racket and the player are also considered.

9. State the law of conservation of momentum.
   a. Momentum is conserved for an isolated system with any number of objects in it.
   b. Momentum is conserved for an isolated system with an even number of objects in it.
   c. Momentum is conserved for an interacting system with any number of objects in it.
   d. Momentum is conserved for an interacting system with an even number of objects in it.

8.3 Elastic and Inelastic Collisions

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Distinguish between elastic and inelastic collisions
- Solve collision problems by applying the law of conservation of momentum

Section Key Terms

elastic collision	inelastic collision	point masses	recoil

Elastic and Inelastic Collisions

When objects collide, they can either stick together or bounce off one another, remaining separate. In this section, we'll cover these two different types of collisions, first in one dimension and then in two dimensions.

In an **elastic collision**, the objects separate after impact and don't lose any of their kinetic energy. Kinetic energy is the energy of motion and is covered in detail elsewhere. The law of conservation of momentum is very useful here, and it can be used whenever the net external force on a system is zero. Figure 8.6 shows an elastic collision where momentum is conserved.