Using mathematical models

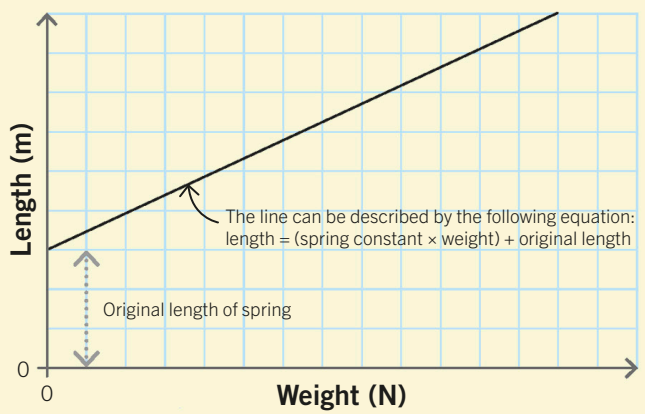
Mathematical models use equations to represent what happens in the real world. Sometimes we can work out a mathematical model from a graph of results. At other times, we might use an equation to predict a result.

Key facts

- ✓ **Mathematical models use equations to represent what happens in the real world.**
- ✓ **Mathematical models can be used to describe graphs of results.**
- ✓ **We can rearrange equations to calculate particular quantities.**

Linear equations

If a relationship between two variables produces a straight line on a graph, we call the relationship linear. Linear relationships can be described by equations written like this: $y = mx + b$. For instance, this graph shows how the length of a spring changes when different weights are hung on it. If you know the original length of the spring and the slope of the line, you can use the graph or the equation to work out the spring's length for any weight.

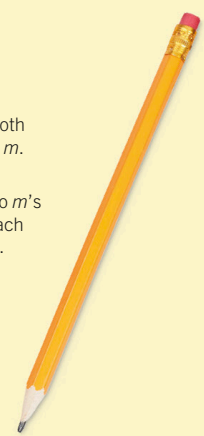


Rearranging equations

Sometimes you need to rearrange an equation before doing a calculation. For example, the equation $F = m \times a$ tells you how to calculate force if you know mass and acceleration, but what if you're told the force and asked to calculate acceleration? You need to rearrange the formula so that a is the subject. You can do it by dividing each side by m. Remember that equations have to stay balanced, so the same operation must be carried out on both sides.

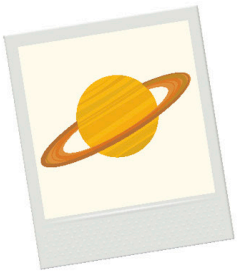
$$F = m \times a$$

1. $F = m \times a$
2. $\frac{F}{m} = \frac{m \times a}{m}$ — Divide both sides by m.
3. $\frac{F}{m} = \frac{\cancel{m} \times a}{\cancel{m}}$ — These two m's cancel each other out.
4. $\frac{F}{m} = a$
5. $a = \frac{F}{m}$



Standard form

Saturn is about 1 400 000 000 000 meters from the Sun. A bacterium is about 0.000 001 meters wide. It's easy to make mistakes in calculations when numbers have lots of zeros, so we simplify them by writing them in "standard form." This shows a long number as a much shorter number (from 1 to under 10) multiplied by a power of 10. To work out the power of 10, count how many times the decimal point has to move.



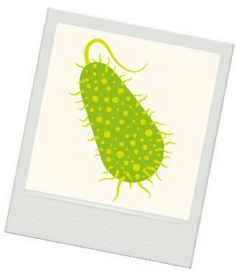
This shows how many places the decimal point has moved.

12 11 10 9 8 7 6 5 4 3 2 1

The second part is a power of 10.

$$1.400\,000\,000\,000 \text{ m} = 1.4 \times 10^{12} \text{ m}$$

The first part is a number with only one digit in front of the decimal point.



The minus sign shows the decimal point has been moved to the right.

1 2 3 4 5 6

$$0.000\,001 \text{ m} = 1.0 \times 10^{-6} \text{ m}$$

Calculating percentages

A percentage is a number shown as a fraction of 100. To turn any fraction into a percentage, work out the fraction on a calculator, multiply the answer by 100, and add a percentage symbol. For example, a 30-watt light bulb transfers 18 watts of power to light and wastes the other 12 watts as heat. What's its efficiency as a percentage?

$$\text{efficiency (\%)} = \frac{\text{useful power output (W)}}{\text{total power input (W)}} \times 100$$

$$= \frac{18 \text{ W}}{30 \text{ W}} \times 100$$

$$= 0.6 \times 100$$

$$= 60\%$$